Angle Measurement

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An *angle* is defined as the figure formed by two lines or planes that intersect one another. Such lines or planes may be real, such as the edges and surfaces of a object, or they may be defined, such as lines from an observer to two distant stars.

The units of measurement of angle are degrees (°) (1° is defined as 1/360 of a circle), and radians (rad) (1 rad is defined as 1/(2 π) of a circle). One radian is equal to 57.29578°, and small angles may be expressed in the unit of milliradians (1×10^{-3} rad). A degree of angle is further divided into 60′ (minutes), and 1′ of angle is divided into 60″ (seconds). One second of angle, being 1/1,296,000 of a circle, is a very small unit of measurement when related to manufactured parts, but is a significant unit when related to much larger dimensions such as the Earth (1″ of angle equals approximately 30 m of a great circle), or in space travel (an included angle of 1″ represents about 9 km on the surface of the moon when it is observed from the Earth during its closest approach to Earth.)

Many terms are used to describe angles in many different fields of expertise. Table 14.1 lists some of these terms, along with very basic definitions.

Many devices and instruments are used to measure or set angles. The following paragraphs describe the variety of equipment involved. See Table 14.2 for a partial list of manufacturers and suppliers of this equipment. See Table 14.3 for a partial list of specific models and approximate prices of angle measurement devices and systems.

14.1 Angle Gage Blocks

Angle gage blocks are right triangle-shaped, hardened and ground steel, flat, about 7 mm thick and 60 mm long. They are supplied in sets that include blocks with one of the acute angles equal to 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, or 30°. These blocks can be used in combination to set work pieces or measure objects in 1° increments. Other sets include blocks with as small as 1″ steps. Special angle gage blocks can be made to any acute angle with the aid of a sine bar and thickness gage blocks.

Angle gage blocks provide a durable, simple, and inexpensive method for measuring and setting angles; for example, positioning work pieces in a vice or fixture prior to a machining operation such as milling, drilling, or grinding.

Because they are not adjustable and made of hardened steel, their accuracy can be assumed by simple observation of their physical condition. Look for indications of wear, nicks, or dents before using.

TABLE 14.1 Defining Terms Relating to Angles

Term	Definition
Angle	A figure formed by two lines or planes that intersect one another.
Acute angle	An angle less than 90°.
Azimuth	The horizontal angle measured along the Earth's horizon, between a fixed reference (usually due south) and an object.
Bank	A lateral inclination.
Circle	A closed plane curve where all of its points are the same distance from its center point.
Declination = declivity	A negative slope.
Degree	Equal to 1/360 of a circle.
Goniometer	An instrument for measuring angles (from the Greek word *gonio*).
Incline = Slope = Bias = Slant = Gradient = Grade	The deviation, plus or minus, from horizontal as defined by gravity.
Latitude	An angle measured north or south from the equator on a meridian to a point on the earth.
Lean = List = Tilt	The deviation from vertical as defined by gravity.
Longitude	The angle between the prime meridian (through Greenwich, England) and the meridian of a given place on Earth. This angle is defined as positive moving west.
Milliradian	An angle equal to 1/1000 rad.
Minute	An angle equal to 1/60°.
Oblique angle	An obtuse or acute angle.
Obtuse angle	An angle greater than 90°.
Quadrant	One quarter of a circle (90°).
Radian	The angle subtended by an arc of a circle equal to the radius of that circle. One radian is equal to 57.29578°.
Rake	Equals the deviation in degrees from being perpendicular (90°) to a line or a plane.
Right angle	An angle of 90°.
Rise	A positive incline.
Second	An angle equal to 1/60′ (1/3600°).
Straight	An angle equal to 180°.
Taper	The change in diameter or thickness per unit length of axis.
Twist	The angle of turn per unit length of axis, as in a gun barrel or a screw thread.

14.2 Clinometers

A *clinometer* is an electronic device that measures vertical angle with respect to gravitational level. It is rectangular, with each side being a 90° to its adjacent sides. With a range of readings of at least ±45°, this shape allows measurements up to a full 360°. Floating zero can be set anywhere and resolutions of ±0.01° are obtainable. Some models will convert readings to inches per foot, % of grade, and millimeters per meter (mm m^{-1}).

A clinometer can be used anywhere the angle of a surface with respect to gravity or another surface needs to be measured. High accuracy and resolution are obtainable, but calibration should be checked periodically with respect to a known level surface and a known angle. Surfaces that are remote to one another or have an intervening obstruction pose no problem for a clinometer.

14.3 Optical Comparator

An *optical comparator* measures angles, along with other dimensions or profiles, by referencing a visual image (usually magnified) of an object to a reticule that is calibrated in the measurement units desired. A hand-held optical comparator is placed directly over the object to be measured and the operator's eye is moved to the proper distance above the comparator for good focus of the magnified image. Some models contain a battery- or line-powered light source. Reticules for these hand-held devices are generally graduated in 1° increments.

TABLE 14.2 A Partial List of Manufacturers and Supplies of Angle Measurement Equipment

Company	Address
Flexbar Machine Corporation (Representative for Erich Preissr & Co., West Germany)	250 Gibbs Road Islandia, NY 11722-2697 Tel: (800) 879-7575
Fred V. Fowler Co., Inc.	66 Rowe Street P.O. Box 299 Newton, MA 02166 Tel: (617) 332-7004
L. S. Starrett Company	121 Crescent Street Athol, MA 01331 Tel: See local distributor
Brown & Sharpe Mfg. Co.	931 Oakton Street Elk Grove Village, IL 60007 Tel: (312) 593-5950
Swiss Precision Instruments, Inc. SPI	2206 Lively Blvd. Elk Grove Village, IL 60007 Tel: (708) 981-1300
Edmund Scientific Co.	101 East Gloucester Pike Barrington, NJ 08007-1380 Tel: (609) 573-6250

Projection-type optical comparators are available as bench or floor models and are made for either horizontal or vertical beam viewing. They use a high-intensity light source and magnifying optics to display an image of an object onto a rear-projection, frosted glass screen that is inscribed with angular as well as linear markings. The image displayed is the result of light being projected past the object, referred to as a shadow graph, or of light being reflected off the surface of the object. The method used is determined by the shape of the viewed object and its surface quality.

Magnification of the optical system in these devices can range from 10× by 100×, with screen diameters ranging from 0.3 m to 1 m.

These instruments are useful for measuring profiles of parts after final matching for the purpose of quality control or duplication.

As the name implies, the image that is projected can be superimposed on a mask or outline drawing placed directly on the view screen so that any deviations from the required shape can easily be determined. Optical comparators are heavy, nonportable devices that require a fairly high amount of maintenance and are best used in a fairly dark room.

14.4 Protractor

A *protractor* is an instrument used for measuring and constructing angles. A direct-reading protractor usually is graduated in 1° increments and can be semicircular or circular in shape. The simplest models are of one-piece construction and made from metal or plastic. Other models include a blade or pointer pivoted in the center of the graduated circle.

More precise protractors are equipped with a vernier scale that allows an angle to be indicated to 5′ of arc. See Figure 14.1 for an explanation of how to read such a vernier scale.

TABLE 14.3 Instruments and Devices Used to Measure or Indicate Angles

Type	Manufacturer	Model	Description	Approx. price
Sine bar	Flexbar	16292	5 in. × 15/16 in. wide	$130.00
		16293	10 in. × 1 in. wide	
		16294	5 in. × 2 in. wide	
		12202	5 in. × 1 in. wide, economy	$30.00
	Fowler	52-455-010	5 in. center to center, 15/16 in. wide	
		52-455-015	10 in. C. to C., 1 in. wide	
		52-455-030	2.5 in. C. to C., 1 in. wide	
	SPI	30-712-4	10 in. C. to C., universal bench center	$3048.00
		98-379-1	5 in. C. to C., 1 in. wide, accuracy between rolls = 0.0003 in.	$31.00
		30-091-3	10 in. C. to C., 1 in. wide, accuracy between rolls = 0.0001 in.	$203.00
	Brown & Sharpe	598-291-121-1	5 in. C. to C., 1 in. wide	
		598-293-121-1	10 in. C. to C., 1 1/8 in. wide	
Sine plate	Flexbar	14612	5 in. C. to C., 6 in. × 3 in. × 2 in.	$320.00
		14615	10 in. C. to C., 12 in. × 6 in. × 2 5/8 in.	$1000.00
	Fowler	57-374-001	5 in. C. to C., 6 in. × 3 in. × 2 in.	
		57-374-004	10 in. C. to C., 12 in. × 6 in. × 2 5/8 in.	
	SPI	77-026-3	10 in. C. to C., 12 in. × 6 in. × 2 5/8 in.	$872.00
	Brown & Sharpe	599-925-10	10 in. C. to C., 12 in. × 6 in. × 2 3/8 in.	
Compound sine plate	Flexbar	14616	5 in. C. to C., 6 in. × 6 in. × 3 1/8 in.	$1100.00
	Fowler	57-375-001	5 in. C. to C., 6 in. × 6 in. × 3 1/8 in.	
	SPI	7-072-7	5 in. C. to C., 6 in. × 6 in. × 3 1/8 in.	$926.00
	Brown & Sharpe	599-926-5	5 in. C. to C., 6 in. × 6 in. × 3 1/2 in.	
Angle Computer	Flexbar	19860	3-axis with vernier protractors	$3750.00
Protractor-Direct	Flexbar	16337	Rectangular Head, 0–180°	$25.00
	Starrett	RP1224W	Head only, To fit 12 in., 18 in. & 24 in. blades	
		C183	Rectangular head, 0–180° 6 in. Blade	
	SPI	30-393-3	Rectangular head, 0–180°	$23.00
		31-804-8	Head only. To fit 12 in., 18 in. & 24 in. Blades	$39.00
Protractor-Vernier	Flexbar	16339	360° range, 1′ reading with magnifier, 12 in. & 6 in. blades incl.	$400.00
		16338	360° range, 5′ reading	$75.00
	Starrett	C364DZ	12 in. Blade, 0–90° range thru 360°, 5′ graduations	
	SPI	30-395-8	6 in. Blade, 0–90° range thru 360°, 5′ graduations	$65.00
		30-390-9	6 in. & 12 in. Blades, 0–90° range thru 360°, 1′ graduations with magnifier	$540.00
	Brown & Sharpe	599-490-8	8 in. Blade, 0–90° range thru 360°, Magnifier optional	
Protractor, Digital (Inclinometer)	Flexbar	17557	±45° range, ±0.1° resolution	$260.00
		17556	±60°, ±0.01° resolution, SPC output	$450.00
	Fowler	54-635-600	±45° range, ±0.01° resolution, RS232 output available	
	SPI	31-040-9	±45° range, resolution: ±0.01° (0 to ±10°), 0.1° (10° to 90°)	$329.00
Protractor, Dial Bevel		30-150-7	8 in. Blade, 1 3/8 in. diameter dial, geared to direct read to 5′	$527.30
	Brown & Sharpe	599-4977-8	8 in. Blade, dial read degrees and 5′	
Square-Reference	SPI	30-392-5	90° fixed angle	$55.00
Optical Comparator (Projector)	Fowler	53-912-000	12 in. screen diameter, 10×, 20×, 25× lens available, horizontal beam, with separate light source for surface illumination	
	Starrett	HB350	14 in. screen diameter, 10×, 20×, 25×, 31.25×, 50×, 100× lens available, horizontal beam	

TABLE 14.3 (continued) Instruments and Devices Used to Measure or Indicate Angles

Type	Manufacturer	Model	Description	Approx. price
		VB300	12 in. screen diameter, 10× through 100× lens available, Vertical beam	
		HS1000	40 in. screen diameter, 10× thru 100× lens available, Horizontal beam	
	SPI	40-350-1	14 in. screen diameter, 10×, 20×, 50× lens available, Horizontal beam	$2995.00
Optical Comparator (hand-held)	SPI	40-145-3	10× Magnification, Pocket style	$57.50
			Additional Reticles	$11.00
		40-140-6	7× Magnification, pocket style with illuminator	$62.50
			Additional Reticles	$10.50
	Edmund Scientific	A2046	6× Magnification, pocket style, 360° protractor reticle, 1° increments	$58.75
Angle Plate	Fowler	52-456-000	Set of 2, 9/32 in. thick, steel, 30 × 60 × 90°, 45 × 45 × 90°	
	SPI	98-463	Set of 2, 5/16 in. thick, steel, 30 × 60 × 90°, 45 × 45 × 90°	$32.00
Angle Positioning Block	SPI	70-997-2	0 to 60°, 10′ vernier (for setting workpiece in a vice)	$122.00
Angle Gage	Fowler	52-470-180	18 leaves, spring steel, 1 thru 10, 14, 14.5, 15, 20, 25, 30, 35, and 45°	
	SPI	31-375-9	18 gage set, 5° thru 90° in 5° steps, 5′ accuracy	$49.60
Angle Gage Blocks	Starrett	Ag18.TR	18 block set, use in combination for steps of 1″, 1″ accuracy	
	Starrett AG16.LM		16 block set, use in combination for steps of 1″, 1/4″ accuracy	
	SPI	30-140-8	10 block set, 1, 2, 3, 4, 5, 10, 15, 20, 25, 30°, Accuracy = ±0.0001″ per inch.	$170.00
			1/4° and 1/2° blocks optional (each)	$18.00

14.5 Sine Bar

A *sine bar* is a device used to accurately measure angles or to position work pieces prior to grinding and other machining procedures. It is constructed from a precisely hardened and ground steel rectangular bar to which are attached two hardened and ground steel cylindrical rods of the same diameter. The axis of each rod is very accurately positioned parallel to the other and to the top surface of the bar.

A sine bar is used in conjunction with precision gage blocks that are placed under one of the two cylindrical rods to raise that rod above the other rod a distance H (see Figure 14.2) equal to the sine of the angle desired, times the distance D between the two rods. The standard distance between the rods is 250 mm (5 in.) or 500 mm (10 in.). The governing equation in using a sine bar is $\sin A = H/D$.

A work piece positioned using a sine bar is usually secured with the use of a precision vice. The vice may clamp directly to the work piece or, when using a sine bar that has tapped holes on its top surface, to the sine bar sides with the work piece bolted to the top of the sine bar.

14.6 Sine Plate

A variation of the sine bar is the *sine plate.* A sine plate consists of the three elements of a sine bar plus a bottom plate and side straps used to lock the plate in the desired position. In addition, one of the ground steel rods is arranged to form a hinge between the top and bottom plates. When using a sine plate, a work piece is secured to the top plate using bolts or clamps and the bottom plate is secured to a machine tool table using clamps or a magnetic chuck.

Compound sine plates are bidirectional, allowing angles to be measured and set in each of two orthogonal planes (true compound angles).

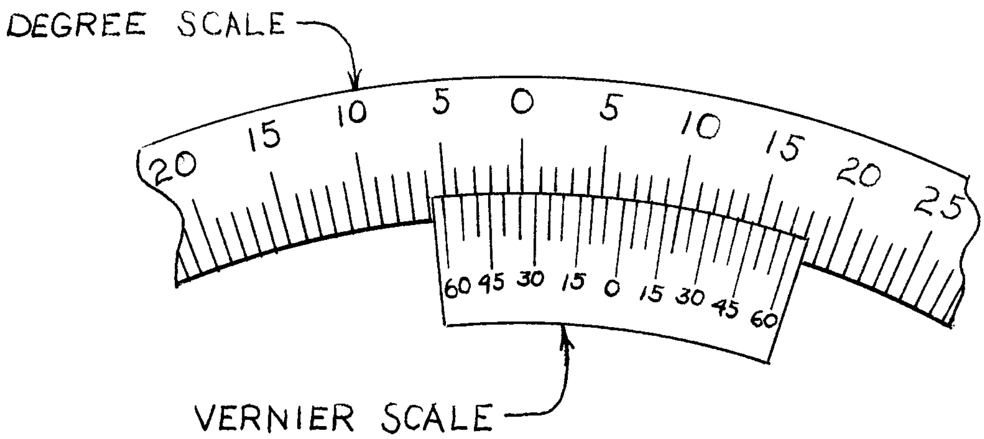

FIGURE 14.1 Vernier protractor. If the zero mark on the vernier scale is to the right of the zero mark on the main scale, as shown in this drawing, then the right side of the vernier scale must be used. Look for the mark on the vernier scale that best aligns with one of the marks of the protractor degree scale. Count the number of marks on the vernier scale from the zero mark to this mark. Each mark thus counted is, in this example, equal to 5′ of arc and, therefore, the number of minutes to be added to the number of degrees indicated is 5 times the vernier marks counted. (In this example, the fourth mark aligns the best with the main scale indicating 20′). The number of degree indicated is the degree mark just to the left of the zero mark on the vernier scale. The left side of the vernier is similarly used when the indicated angle is to the left of the zero mark on the degree scale.

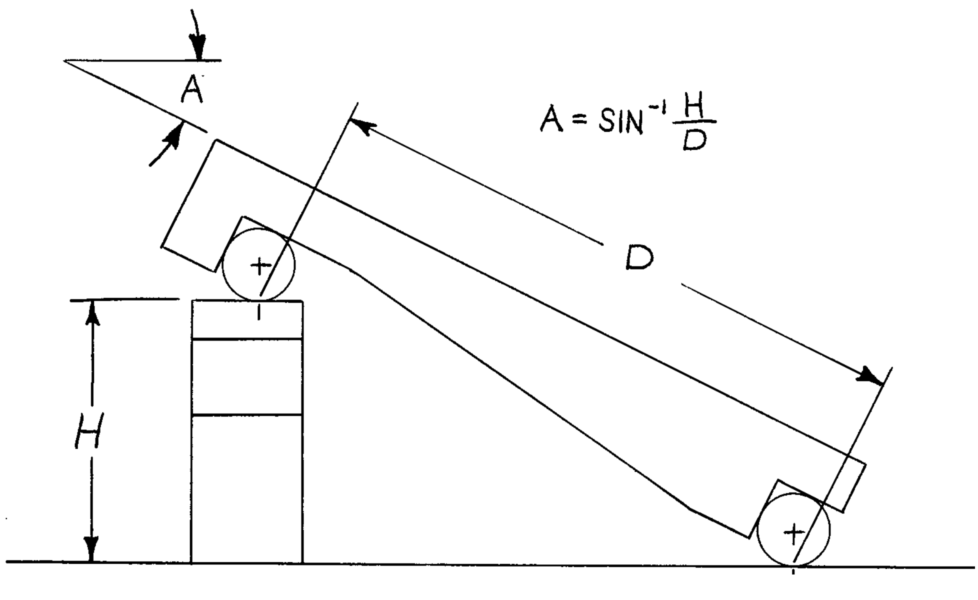

FIGURE 14.2 Sine bar. A sine bar is used in conjunction with precision gage blocks that are placed under one of the cylindrical rods, raising that end a distance, *H*, equal to the sine of the desired angle times the distance, *D*, between the two rods.

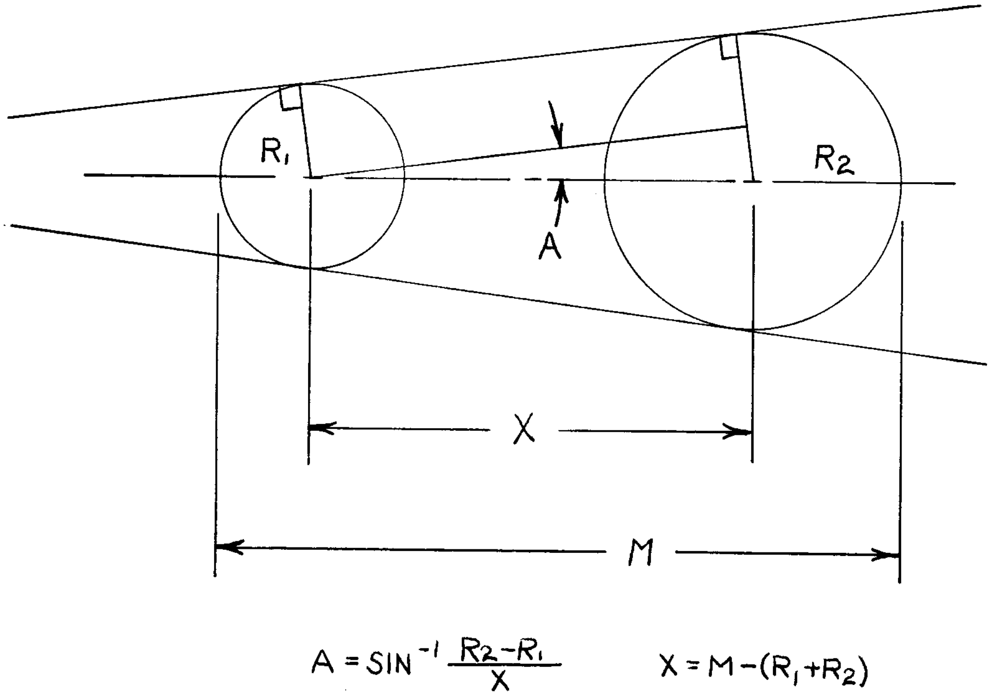

FIGURE 14.3 Measuring tapers. Two balls (for holes) or gage pins (for slots) should be selected to fit the hole size to be measured. The distance between the balls or pins should be as large as possible to allow for the best accuracy of measurement. The position of the balls can be determined and the equations shown in this figure applied to determined the angle of the taper.

14.7 Taper

An accurate method that can be used to measure a tapered hole is described in Figure 14.3. In this method, one uses a pair of steel balls of proper diameters to match the hole size and angle of taper. This method can also be used to measure the angle between two adjacent planes, using either balls or gage pins.

Further Information

Further information on this subject can be found in company catalogs in both print and Internet formats. An additional reference is *Machinery's Handbook*, 25th ed., Industrial Press Inc., 200 Madison Avenue, New York, NY 10016-4078, 1996.

Tilt Measurement

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15.1 Tiltmeters or Inclinometers
15.2 Geodetic Leveling
15.3 Hydrostatic Leveling
15.4 Suspended and Inverted Plumb Lines
15.5 Integration of Observations

If the relative position of two points is described by the three-dimensional orientation of a line joining them, then, in general, *tilt* is the angular amount that the orientation has changed, in a vertical plane, from a previous direction or from a reference direction. If the original or reference orientation is nearly horizontal, then the term "tilt" is usually used. If it is nearly vertical, then the change is often regarded as "inclination." Here, "tilt" will refer to either. The two points can be separated by a discernable amount, the base, or the tilt can be measured at a point with the reference orientation being defined by the direction of the force of gravity at that point. Thus, the same instrument that measures tilt at a point can be called either a *tiltmeter* or an *inclinometer* (or clinometer), depending on the interpretation of the results. The instrument used to measure a series of tilts along any vertical orientation is often called an inclinometer (e.g., Dunnicliff [1]).

Angular tilt is directly related to the linear amount of change subtending the length of the base. Consequently, angular tilt does not have to be measured directly but can be derived from the mechanical or other measurement of this linear change if the length of the base is known.

Therefore, the following discussion has been subdivided into:

1. Tiltmeters or inclinometers (angular tilt at a point or over a limited, relatively small base length)
2. Geodetic leveling (tilt derived from a height difference over an extended base of virtually limitless length)
3. Hydrostatic leveling (tilt derived from a height difference over an extended base of limited length)
4. Suspended and inverted pendula, or plumb lines (inclination from a difference in horizontal relative position over a vertical base or height difference)

15.1 Tiltmeters or Inclinometers

Considering the basic principle of operation, tiltmeters may be divided into: liquid (including spirit bubble type), vertical pendulum, or horizontal pendulum. Dunnicliff [1] provides a comprehensive review of tiltmeters and inclinometers according to the method in which the tilt is sensed (i.e., mechanical, with accelerometer transducer, with vibrating wire transducer, or with electrolytic transducer).

The sensitivity of tilt falls into two distinct groups: geodetic or geophysical special tiltmeters with a resolution of 10^{-8} rad (0.002″) or even 10^{-9} rad; and engineering tiltmeters with resolutions from 0.1″ to several seconds of arc, depending on the required range of tilt to be measured.

The first group includes instruments that are used mainly for geophysical studies of Earth tide phenomena and tectonic movements; for example, the Verbaander-Melchior [2] and the Zöllner [3, 4] horizontal pendulum tiltmeters, and the Rockwell Model TM-1 [5] liquid-bubble type. This category of instrument requires extremely stable mounting and a controlled environment. There are very few engineering projects where such sensitive instruments are required. However, deformation measurements of underground excavations for the storage of nuclear waste may be one of the few possible applications. An example is a mercury tiltmeter (Model 300) developed for that purpose by the Auckland Nuclear Accessory Co. Ltd. in New Zealand. In this instrument, the change in capacitance between electrodes and a mercury surface is proportional to the tilt. This tiltmeter, with a total range of 15″, is claimed to give a resolution of 10^{-9} rad (0.0002″), which corresponds to a relative vertical displacement of only 6×10^{-7} mm over its base length of 587 mm.

In the second group, there are many models of liquid or pendulum tiltmeters of reasonable price ($2000 to $5000) that satisfy most needs in engineering studies. Apart from a spirit level or level vial by itself, the simplest form of tiltmeter is a base that is tens of centimeters long and leveled by centering the bubble in the vial by direct viewing or by an optical coincidence viewing of the two ends of the bubble. Direct viewing gives a resolution of 1/5 of a vial division (usually 2 mm), which typically has a sensitivity of 10″ to 30″ per division. Coincidence viewing increases the setting accuracy to 0.03 of the sensitivity of the vial. The discrepancy from horizontal between the two measurement points can be determined by a dial gage or micrometer that has a resolution of 0.0005 in. or 0.02 mm. Huggenberger AG claim a sensitivity of 0.3″ (1×10^{-4} gon) over a range of ±21′ for their clinometer with a 100 mm base and coincidence centering of the bubble in the level vial. The clinometer can be attached to 1 m bases for either horizontal or vertical measurements.

If the vial is filled with an electrolytic liquid, the centering of the bubble can be done electrically. An example is the Electrolevel (by the British Aircraft Corp.), which uses the spirit bubble principle [6] and in which the movement of the bubble is sensed by three electrodes. A tilt produces a change in differential resistivity between the electrodes that is measured by means of a Wheatstone bridge. A resolution of 0.25″ is obtained over a total range of a few minutes of arc. Many other liquid types of tiltmeters with various ranges (up to 30°) are available from various companies. Holzhausen [7] and Egan and Holzhausen [8] discuss the application of electrolytic tiltmeters (resolution of 2″ over a range of ±1°, manufactured by Applied Geomechanics) in the monitoring of various hydroelectric power dams in the U.S.

The Rank Organization in the United Kingdom [9] makes a liquid-dampened pendulum-type electronic level, the Talyvel, which gives an accuracy of ±0.5″ over a total range of ±8′. A similar transducer of the pendulum type is used in the Niveltronic tiltmeter (range of ±150″ with an accuracy of ±0.2″) produced by Tesa S.A. in Switzerland. Of particular popularity are servo-accelerometer tiltmeters with horizontal pendula. They offer ruggedness, durability, and can operate in low temperatures. The output voltage is proportional to the sine of the angle of tilt. Schaevitz Engineering produces such a servo-accelerometer that employs a small-mass horizontal paddle (pendulum) which tries to move in the direction of tilt, due to the force of gravity. Any resultant motion is converted by position sensors to a signal input to the electronic amplifier whose current output is applied to the torque motor. This develops a torque that is equal and opposite to the original. The torque motor current produces a voltage output that is proportional to the sine of the angle of tilt.

The typical output voltage range for tiltmeters is ±5 V, which corresponds to the maximum range of tilt and readily allows for serial interfacing. The angular resolution of a tiltmeter depends on its range of tilt since a larger range would result in more angular tilt per unit voltage so a higher resolution tiltmeter would have a smaller range of measurable tilt. Typically, the resolution is 0.02% of the range (volts) [10].

There are many factors affecting the accuracy of tilt sensing, not just the resolution of the readout. A temperature change produces dimensional changes in the mechanical components and changes in the viscosity of the liquid in electrolytic tiltmeters and of the dampening oil in pendulum-type tiltmeters. Also, electric characteristics can alter with temperature changes. Drifts in tilt indications and fluctuations of the readout may also occur. Compensation for the effects of temperature changes can be incorporated

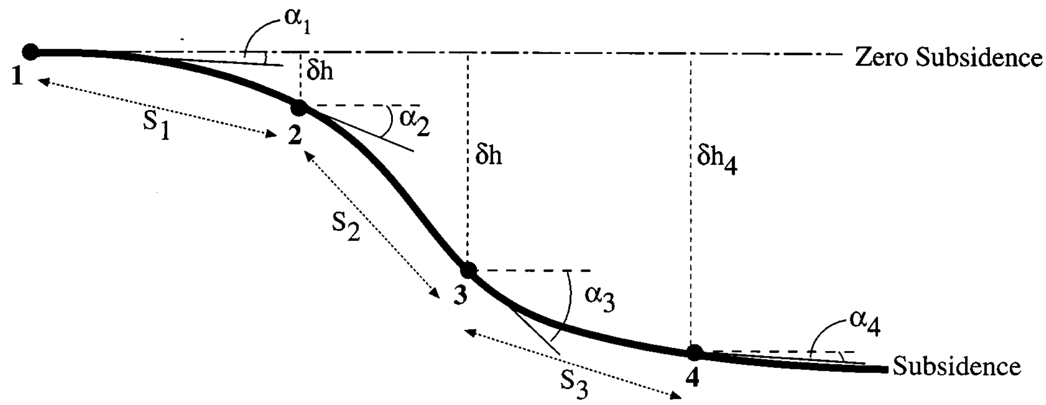

FIGURE 15.1 Ground subsidence derived from tilt measurements.

in the construction of an instrument, but at an increased cost. An alternative is to design a linear reaction by the instrument to the effects of temperature and to apply simple calibration corrections.

In less expensive models, compensation for the aforementioned sources of error is not very sophisticated, and such tiltmeters may show nonlinear output in reaction to changes in temperature and erratic drifts or other behavior that would be difficult to predict without testing. Consequently, very thorough testing and calibration are required even when the accuracy requirement is not very high [11]. Testing should investigate, at least, the linearity of output in reaction to induced tilts over the instrument's full range (±) and to changes in temperature. Some suggestions for testing and calibrating inclinometers, among other instruments, are given in Dunnicliff [1]. It is further emphasized that regular and up-to-date calibration is important in order to ensure continuity in the fidelity of the data being gathered. In most cases, the behavior being investigated changes with time and incorrect data cannot be recaptured.

Compensators for vertical circle readings in precision theodolites work on the same principle as some engineering tiltmeters. The liquid compensator of the Kern E2 electronic theodolite [12] gave a repeatability of better than 0.3″ over a range of ±150″ and was incorporated in their tiltmeter, NIVEL 20, in 1989. The same compensation system has been used in the currently available Leica TC2002 precision electronic total station [13]. Consequently, the theodolite may also be used as a tiltmeter, in some applications, giving the same accuracy as the Electrolevel, for example.

Tiltmeters have a wide range of applications. A series of tiltmeters, if arranged along a terrain profile in a mining area, may replace geodetic leveling in the determination of ground subsidence [11] as shown in Figure 15.1. For example, the subsidence (i.e., the variation from the previous or original profile) of point 4 (δh_4) with respect to point 1 may be estimated from the observed changes in tilt, from a base or original position, (α_i in radians) and known distances between the points as:

$$\delta h_4 = s_1\left(\alpha_1 + \alpha_2\right)/2 + s_2\left(\alpha_2 + \alpha_3\right)/2 + s_3\left(\alpha_3 + \alpha_4\right)/2 \qquad (15.1)$$

The fidelity of this method depends on the density of tilt measurements along the profile and the continuity of the profile (a constant slope of the terrain between measurement points is assumed). Similarly, deformation profiles of tall buildings may be determined by placing a series of tiltmeters at different levels of the structure [14]. Also, changes in borehole profiles can be created in a similar manner. The absolute profile of a borehole can be generated by considering the horizontal displacement in the direction of the orientation of the inclinometer (usually controlled by guide grooves in the borehole casing) for the *i*-th position, as it traverses a borehole with observation of α_i at a depth s_i. However, this would require calibration of the inclinometer to correct its output to show zero in its vertical position since the α_i are tilts from the vertical rather than angular changes from an original inclination.

In geomechanical engineering, the most popular application of tiltmeters and borehole inclinometers is in slope stability studies and in monitoring earth-fill dams. Torpedo-shaped biaxial inclinometers are used to scan boreholes drilled to the depth of an expected stable strata in the slope. By lowering the inclinometer on a cable with marked intervals and taking readings of the inclinometer at those intervals, a full profile of the borehole and its changes may be determined through repeated surveys, as mentioned above. SINCO and other producers of tiltmeters provide special borehole inclinometers (50 cm or 2 ft long) with guide wheels to control the orientation of the inclinometer. A special borehole casing (plastic or aluminum) with guiding grooves for the wheels is available. Usually, servo-accelerometer type inclinometers are used with various ranges of inclination measurements; for example, ±6°, ±53°, or even ±90°. A 40 m deep borehole, if measured every 50 cm with an inclinometer having an accuracy of only ±100″, should allow for the determination of linear lateral displacement of the collar of the borehole with an accuracy of ±2 mm.

In cases where there is difficult access to the monitored area or a need for continuous data acquisition or both, tiltmeters or borehole inclinometers can be left in place at the observing station with a telemetry monitoring system allowing for communication to the processing location. One example of a station setup of a telemetric monitoring of ground subsidence in a mining area near Sparwood, B.C. used a telemetry system developed for the Canadian Centre for Mining and Energy Technology (CANMET) by the University of New Brunswick [11, 15]. Terra Technology biaxial servo-accelerometer tiltmeters of ±1° range were used in the study. The telemetry system could work with up to 256 field stations. Each station accepted up to six sensors (not only tiltmeters but any type of instrument with electric output, e.g., temperature, power level or voltage). Another example is a fully automated borehole scanning system with a SINCO inclinometer and telemetric data acquisition that was also developed at the University of New Brunswick [16]. It has been used successfully in monitoring highwall stability at the Syncrude Canada Limited tarsands mining operation in northern Alberta.

15.2 Geodetic Leveling

Geodetic or differential leveling measures the height difference between two points using precision tilting levels, or precision automatic levels with compensators, with parallel plate micrometers and calibrated invar leveling staves or rods. Recent technology has provided digital automatic levels that use a CCD sensor in the instrument and bar codes, rather than linear graduations, on the staves [17]. Their ease of use and comparable precision have quickly made them rivals to the traditional optical instruments. In a single setup of the level, the height difference is the backsight rod reading minus the foresight rod reading. Any number of setup height differences can be combined to determine the height difference between two points of interest; however, the errors involved accumulate with the number of setups. With sight lengths limited to no more than 20 m, geodetic leveling can produce height differences with a precision of ±0.1 mm per setup, which is equivalent to a precision of ±0.5″ in tilt. Although geodetic leveling is traditionally used to determine elevations, it is often used to monitor not only the settlement of sensitive structures but also to describe the tilt of components of a structure by determining the tilt between appropriate pairs of benchmarks (monumented in or on the structure) [18]. Since the level reference is created by an optical line of sight through a telescope (magnification up to 40×), a major source of systematic error is the effect of vertical atmospheric refraction. A vertical temperature gradient of even 1°C m^{-1} across the line of sight would bend the line of sight to be in error by 0.4 mm at 30 m. Gradients of this magnitude are commonly encountered in industrial settings and are usually even more evident outdoors. Less effect is realized if the sight lengths are kept short, but this must be weighted against the accumulation of random error with each additional setup (shorter sight lengths would require more setups to traverse the same height difference). The errors that seem to be insignificant in a single setup (or in a few setups) become magnified in height differences involving a large number of setups (e.g., rod scale error and settlement of the instrument or rods). Such errors have become quite noticeable in the monitoring of tectonic plate movement and undetected systematic effects can be misleading. Further discussion on precision leveling and sources of error is available in Vanicek et al. [19] and in Schomacker and Berry [20].

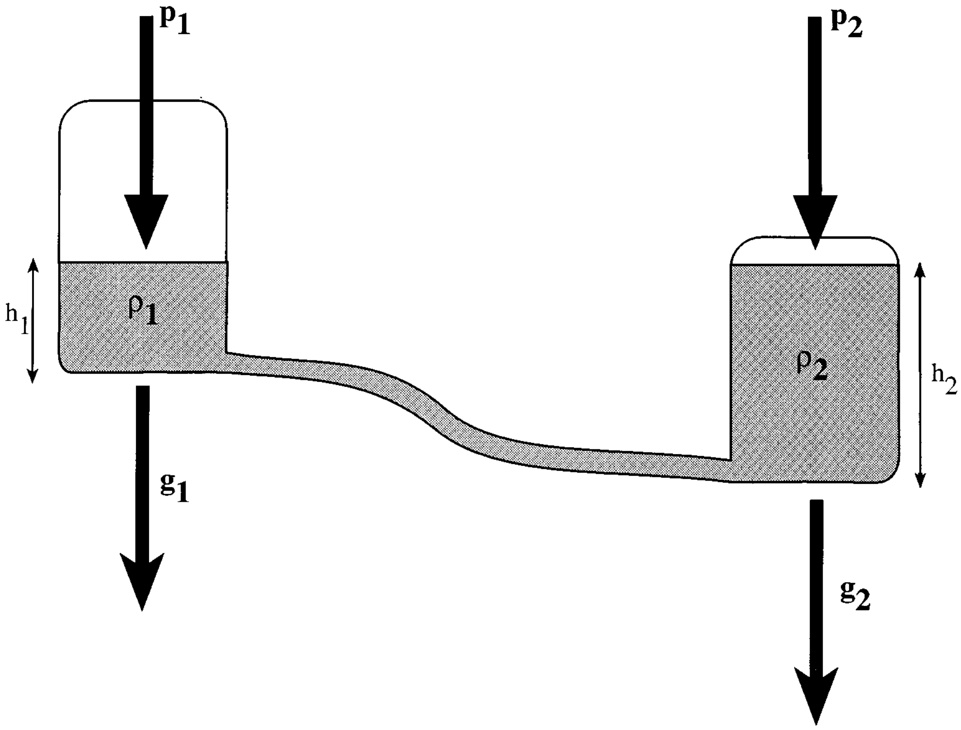

FIGURE 15.2 Hydrostatic equilibrium in connected vessels.

Having the elevations or heights, h_1 and h_2, of two points or having measured, or determined, the height difference between them, $\Delta h_{12^t} = h_2 - h_1$, at a time t, means that, if $\delta\Delta h = \Delta h_{12^{t2}} - \Delta h_{12^{t1}}$, the tilt, T_{12}, can be calculated if the horizontal separation s_{12} is known, since $T_{12} = \delta\Delta h/s_{12}$. The separation does not have to be known as precisely as the height difference since the total random error is $\sigma_{T^2} = \sigma_{\delta\Delta h^2}/s^2 + \sigma_{s^2}(\delta\Delta h^2/s^4)$. As an example, for two points that are 60 m apart with a height difference of 0.5 m (extreme in most structural cases) with the height difference known to ±50 μm ($\sigma_{\delta\Delta h}$) and the distance known to ±0.01 m (σ_s), the tilt would have a precision (σ_T) of ±0.3″. Further, neither the measurement of the height difference nor the determination of the separation have to be done directly between the two points. The leveling can be done along whatever route is convenient and the separation can be obtained in a variety of ways, for example, inversing from coordinated values for the points [21].

15.3 Hydrostatic Leveling

If two connected containers (Figure 15.2) are partially filled with a liquid, then the heights h_1 and h_2 are related through the hydrostatic equation (Bernoulli's equation, as given in [22]):

$$h_1 + P_1/(g_1\rho_1) = h_2 + P_2/(g_2\rho_2) = c \qquad (15.2)$$

where P is the barometric pressure, g is the force of gravity, ρ is the density of the liquid which is a function of temperature, and c is a constant.

The above relationship has been employed in hydrostatic leveling, as shown schematically in Figure 15.3. The air tube connecting the two containers eliminates possible error due to different air pressures at two stations. The temperature of the liquid should also be maintained constant because, for example, a difference of 1.2°C between two containers may cause an error of 0.05 mm in a Δh determination

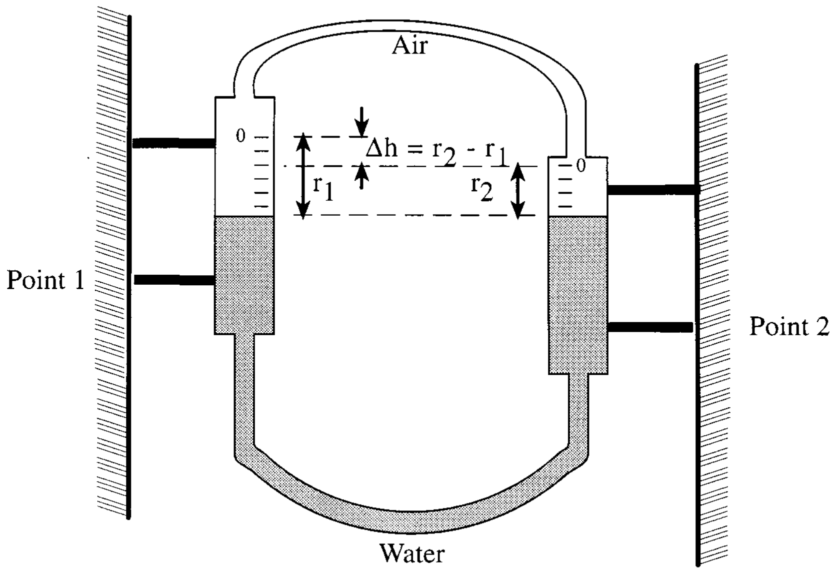

FIGURE 15.3 Hydrostatic leveling ($\Delta h_{12} = r_2 - r_1$).

for an average $h = 0.2$ m and $t = 20°C$. Huggett et al. [23] devised a two-fluid tiltmeter to overcome the temperature effect by using two liquids with different temperature coefficients and claim a resolution of 10^{-8} to 10^{-9} rad over a separation of up to 1 km. In a discussion of liquid level gages, Dunnicliff [1] emphasizes that care should be exercised to ensure that there is no discontinuity in the liquid since any gas (usually air, often entering when filling the tubing) in the liquid line will introduce an error in the level difference, especially in a vertical, more than in a horizontal, portion of tubing. He also mentions that the behavior of the liquid is influenced by the inside diameter and capillary effects of the tubing, while the outside diameter is likely what is quoted by manufacturers. Dunnicliff [1] also provides a comprehensive summary of the variety of liquid level gages.

Two examples of typical hydrostatic instruments used in precision leveling will be mentioned here. The ELWAAG 001, developed in Germany [24], is a fully automatic instrument with a traveling (by means of an electric stepping motor) sensor pin that closes the electric circuit on touching the surface of the liquid. A standard deviation of ±0.03 mm is claimed over distances of 40 m between the instruments [22]. Another automatic system, the Nivomatic Telenivelling System, is available from Telemac or Roctest Ltd. The Nivomatic uses inductance transducers that translate the up and down movements of its floats into electric signals (frequency changes in a resonant circuit). An accuracy of ±0.1 mm is claimed over a 24 m length. P & S Enterprises, Ltd. produces a Pellissier model H5 portable hydrostatic level/tiltmeter, for which they claim an accuracy of ±5 μm over a tube length of 14 m, for engineering and Earth tide measurements.

Hydrostatic levels may be used in a network formation of permanently installed instruments to monitor tilts in large structures. Robotti and Rossini [25] report on a DAG (Automatic Measuring Device of Levels and Inclinations) network monitoring system available from SIS Geotecnica (Italy) that offers an accuracy of about ±0.01 mm using inductive transducers in the measurement of liquid levels. Various systems of double liquid (e.g., water and mercury) settlement gages based on the principle of hydrostatic leveling are used for monitoring power dams [26] with extended networks of connecting tubing.

Instruments with direct measurement of liquid levels are limited in their vertical range by the height of their containers. This problem may be overcome if liquid pressures are measured instead of the changes in elevation of the liquid levels. Pneumatic pressure cells or pressure transducer cells may be used. Both Dunnicliff [1] and Hanna [26] give numerous examples of various settlement gages based on that principle. Meier [27] mentions the application of a differential pressure tiltmeter in the monitoring of a concrete dam.

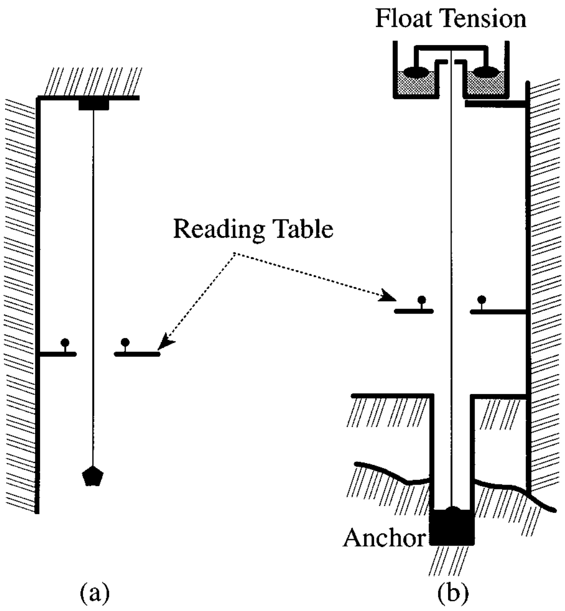

FIGURE 15.4 Inclination measurements with plumblines. (*a*) suspended pendulum; (*b*) inverted pendulum.

15.4 Suspended and Inverted Plumb Lines

Two types of mechanical plumbing are used in monitoring the stability of vertical structures:

1. Suspended pendula or plumb lines (Figure 15.4(*a*))
2. Floating pendula or inverted, or reversed, plumb lines (Figure 15.4(*b*))

Typical applications are in the monitoring of power dams and of the stability of reference survey pillars. Suspended pendula are also commonly used in mine orientation surveys and in monitoring the stability of mine shafts. Tilt, or inclination, is derived from differences in horizontal relative position combined with vertical separation in the same way as tilt is derived from geodetic leveling. So, similarly, the vertical separation between two reading tables or between a reading table and anchor point does not have to be known as precisely as the change in relative position. Two table readings, each ±0.02 mm, with a relative position difference of 100 mm and a vertical separation of 10 m, known to ±0.01 m, would result in a tilt precision of ±2″.

Inverted plumb lines have become standard instrumentation in large dams (e.g., Hydro Quebec uses them routinely). Their advantage over suspended plumb lines is in the possibility of monitoring the absolute displacements of structures with respect to deeply anchored points in the foundation rock which may be considered as stable. In power dams, the depth of anchors must be 30 m or even more below the foundation in order to obtain absolute displacements of the surface points. The main problem with inverted plumb lines is the drilling of vertical boreholes so that the vertical wire of the plumb line would have freedom to remain straight and vertical. A special technique for drilling vertical holes has been developed at Hydro Quebec [28].

Several types of recording devices that measure displacements of structural points with respect to vertical plumb lines are produced by different companies. The simplest are mechanical or electromechanical micrometers with which the plumb wire can be positioned with respect to reference lines of a recording (coordinating) table with an accuracy of ±0.2 mm or better. Traveling microscopes may give the same accuracy. Automatic sensing and recording is possible with a Telecoordinator from Huggenberger

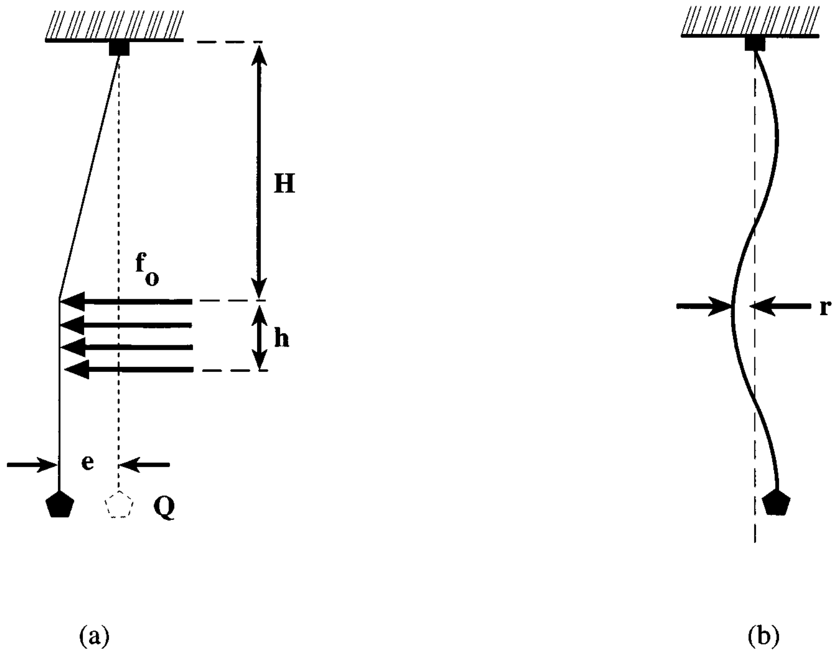

FIGURE 15.5 (*a*) Influence of air currents on a suspended plumbline. (*b*) Horizontal error due to the spiral shape of the wire.

AG in Switzerland. Telemac Co. (France) developed a system, Telependulum (marketed by Roctest), for continuous sensing of the position of the wire with remote reading and recording. A rigidly mounted reading table supports two pairs of induction type proximity sensors arranged on two mutually perpendicular axes. A hollow cylinder is fixed on the pendulum wire at the appropriate level, passing through the center of the table and between the sensors. Changes in the width of the gap between the target cylinder and the sensors are detected by the corresponding changes in the induction effect. The system has a resolution of ±0.01 mm.

An interesting Automated Vision System has been developed by Spectron Engineering (Denver, Colorado). The system uses solid state electronic cameras to image the plumb line with a resolution of about 3 μm over a range of about 75 mm. Several plumb lines at Glen Canyon dam and at Monticello dam, near Sacramento, California, have been using the system since 1982 [29].

Two sources of error, which may be often underestimated by the user, may strongly affect plumb line measurements:

1. The influence of air currents
2. The spiral shape of the wire

If the wire of a plumb line (Figure 15.5(*a*)), with pendulum mass Q, is exposed along a length h to an air current of speed v at a distance H from the anchor point, then the plumb line is deflected by an amount [30]:

$$e = f_o hH/Q \tag{15.3}$$

where f_o is the acting force of air current per unit length of the wire. The value of f_o may be calculated approximately from [30]

$$f_o = 0.08\,dv^2/Q \tag{15.4}$$

where d is the diameter of the wire in millimeters, v is in meters per second, and Q is in kilograms. As an example, if $H = 50$ m, $h = 5$ m, $d = 1$ mm, $Q = 20$ kg, and $v = 1$ m s^{-1} (only 3.6 km h^{-1}) then $e = 1$ mm.

The second source of error, which is usually underestimated in practice, is that the spiral shape (annealing) of the wire (Figure 15.5(*b*)) affects all wires unless they are specially straightened or suspended for a prolonged time (on the order of several months). If the wire changes its position (rotates) between two campaigns of measurements, then the recorded displacements could have a maximum error of $2r$. The value of r can be calculated from [30]:

$$r = \left(\pi d^4 E\right) \Big/ \left(64 RQ\right) \tag{15.5}$$

where E is Young's modulus of elasticity (about 2×10^{11} Pa for steel); R is the radius of the free spirals of the unloaded wire that, typically, is about 15 cm for wires up to 1.5 mm diameter; and d and Q are as above. For a plumb wire with $d = 1$ mm, $R = 15$ cm, and $Q = 196$ N (i.e., 20 kg), $r = 0.3$ mm.

If one plumb line cannot be established through all levels of a monitored structure, then a combination of suspended and inverted plumb lines may be used as long as they overlap at least at one level of the structure. At Hydro Quebec, the drill holes of the plumb lines are also used for monitoring height changes (vertical extension) by installing tensioned invar wires [31].

15.5 Integration of Observations

The above discussion has considered using individual instruments. Because many investigations, using other instrumentation as well as the measurement of tilt, involve the repetition of measurements, often over a long term, the fidelity of the measurements and their being referenced to the original or initial observation is vital to the investigation. It is risky to expect consistent behavior of instrumentation, particularly in environments with dramatic variations (especially in temperature), and over a long period of time. Any conclusion relating to the behavior of a structure is only as good as the data used in the analysis of the behavior. Two ways to ensure reliability are possible. One is to make regular testing and calibration a component of the observation regimen. The other is to analyze the observations as they are accumulated, either each observable as a temporal series of repeated measurements or observations of different locations or types together in an integrated analysis. The analytical tools for integrated analyses, as well as for calibration testing and temporal series analysis, have been developed [21, 32] and successfully implemented in several projects [15, 18, 33]. Proper calibration testing and correction, rigorous statistical analysis of trend in temporal series, and integrated analysis have proven to be valuable tools in the analysis of deformations and serve to enhance the monitoring of sensitive structures.

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Appendix

A Sampling of Possible Suppliers of Tilt Measuring Instrumentation	
Applied Geomechanics Inc. 1336 Brommer Street Santa Cruz, CA 95062	Auckland Nuclear Accessory Company Ltd. P.O. Box 16066 Auckland, 3. New Zealand
Eastman Whipstock GmbH Gutenbergstrasse 3 3005 Hannover-Westerfeld West Germany	Geotechnical Instruments Ltd. Station House, Old Warwich Road Leamington Spa, Warwickshire CV31 3HR England
Huggenberger AG Holstrasse 176 CH-8040 Zürich Switzerland	IRAD GAGE Etna Road Lebanon, NH 03766
Leica AG CH-9435 Heerbrugg Switzerland	Measurement Devices Limited 11211 Richmond Avenue, Suite 106, Building B Houston, TX 77082
Maihak AG Semperstrasse 38 D-2000 Hamburg 60 West Germany	Roctest Ltée Ltd. 665 Pine Montreal, P.Q. Canada J4P 2P4
RST Instruments Ltd. 1780 McLean Avenue Port Coquitlam, B.C. Canada V3C 4K9	Schaevitz Engineering P.O. Box 505 Camden, NJ 08101
Serata Geomechanics, Inc. 4124 Lakeside Drive Richmond, CA 94806	SINCO (Slope Indicator Co.) 3668 Albion Place N. Seattle, WA 98103
Soil Instruments Ltd. Bell Lane, Uckfield East Sussex TN22 1Ql England	Solexperts AG Postfach 274 CH-8034 Zürich Switzerland
Solinst Canada Ltd. 2440 Industrial St. Burlington, Ontario Canada L7P 1A5	SIS Geotecnica Via Roma, 15 20090 Segrate (Mi) Italy
Spectron Engineering 800 West 9th Avenue Denver, CO 80204	Spectron Glass and Electronics Inc. 595 Old Willets Path Hauppauge, NY 11788
Telemac 2 Rue Auguste Thomas 92600 Asnieres France	Terrametrics 16027 West 5th Avenue Golden, CO 80401
P & S Enterprises, Ltd. 240 South Monaco Pkwy, # 302 Denver, CO 80224	Edi Meier & Partner 8408 Winterthur Switzerland