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Preface

Introduction

The purpose of *Measurement, Instrumentation, and Sensors Handbook CRCnetBase 1999* is to provide a reference that is both concise and useful for engineers in industry, scientists, designers, managers, research personnel and students, as well as many others who have measurement problems. The *CD-ROM* covers an extensive range of topics that comprise the subject of measurement, instrumentation, and sensors.

The *CD-ROM* describes the use of instruments and techniques for practical measurements required in engineering, physics, chemistry, and the life sciences. It includes sensors, techniques, hardware, and software. It also includes information processing systems, automatic data acquisition, reduction and analysis and their incorporation for control purposes.

Articles include descriptive information for professionals , students ,and workers interested in measurement. Articles include equations to assist engineers and scientists who seek to discover applications and solve problems that arise in fields not in their specialty. They include specialized information needed by informed specialists who seek to learn advanced applications of the subject, evaluative opinions, and possible areas for future study. Thus, the *CD-ROM* serves the reference needs of the broadest group of users — from the advanced high school science student to industrial and university professionals.

Organization

The *CD-ROM* is organized according to the *measurement problem*. Section I includes general instrumentation topics, such as accuracy and standards. Section II covers spatial variables, such as displacement and position. Section III includes time and frequency. Section IV covers solid mechanical variables such as mass and strain. Section V comprises fluid mechanical variables such as pressure, flow, and velocity. Section VI covers thermal mechanical variables such as temperature and heat flux. Section VII includes electromagnetic variables such as voltage and capacitance. Section VIII covers optical variables such as photometry and image sensors. Section IX includes radiation such as x rays and dosimetry. Section X covers chemical variables in composition and environmental measurements. Section XI includes biomedical variables such as blood flow and medical imaging. Section XII comprises signal processing such as amplifiers and computers. Section XIII covers display such as cathode ray tube and recorder. Section XIV includes control such as optimal control and motion control. The Appendix contains conversion factors to SI units.

Locating Your Topic

To find out how to measure a given variable, do a word or phrase search, select the section and the chapters that describe different methods of making the measurement. Consider the alternative methods of making the measurement and each of their advantages and disadvantages. Select a method, sensor, and signal processing method. Many articles list a number of vendors to contact for more information. You can also visit the http://www.sensorsmag.com site under Buyer's Guide to obtain a list of vendors.

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I appreciate the help of the many people who worked on this handbook. David Beams assisted me by searching books, journals, and the Web for all types of measurements, then helped me to organize the outline. The Advisory Board made suggestions for revision and suggested many of the authors. Searching the INSPEC database yielded other authors who had published on a measurement method. At CRC Press, Felicia Shapiro, Associate Production Manager;Kristen Maus, Developmental Editor; Suzanne Lassandro, Book Group Production Director; and Susan Fox, Project Editor, produced the book.

John G. Webster
Editor-in-Chief

Editor-in-Chief

John G. Webster received the B.E.E. degree from Cornell University, Ithaca, NY, in 1953, and the M.S.E.E. and Ph.D. degrees from the University of Rochester, Rochester, NY, in 1965 and 1967, respectively.

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Contents

XIII Displays

XIV Control

Characteristics of Instrumentation

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In addressing measurement problems, it is often useful to have a conceptual model of the measurement process. This chapter presents some of the fundamental concepts of measurement in the context of a simple generalized instrument model.

In abstract terms, an *instrument* is a device that transforms a *physical variable* of interest (the *measurand*) into a form that is suitable for recording (the *measurement*). In order for the measurement to have broad and consistent meaning, it is common to employ a standard system of *units* by which the measurement from one instrument can be compared with the measurement of another.

An example of a basic instrument is a ruler. In this case the measurand is the length of some object and the measurement is the number of units (meters, inches, etc.) that represent the length.

1.1 Simple Instrument Model

Figure 1.1 presents a generalized model of a simple instrument. The physical process to be measured is in the left of the figure and the measurand is represented by an observable physical variable ***X***. Note that the observable variable ***X*** need not necessarily be the measurand but simply related to the measurand in some known way. For example, the mass of an object is often measured by the process of *weighing,* where the measurand is the mass but the physical measurement variable is the downward force the mass exerts in the Earth's gravitational field. There are many possible physical measurement variables. A few are shown in Table 1.1.

The key functional element of the instrument model shown in Figure 1.1 is the *sensor,* which has the function of converting the *physical variable input* into a *signal variable output.* Signal variables have the property that they can be manipulated in a transmission system, such as an electrical or mechanical circuit. Because of this property, the signal variable can be transmitted to an output or recording device that can be remote from the sensor. In electrical circuits, voltage is a common signal variable. In mechanical systems, displacement or force are commonly used as signal variables. Other examples of signal variable are shown in Table 1.1. The signal output from the sensor can be displayed, recorded, or used as an input signal to some secondary device or system. In a basic instrument, the signal is transmitted to a *display* or recording device where the measurement can be read by a human observer. The observed output is the measurement ***M***. There are many types of display devices, ranging from simple scales and dial gages to sophisticated computer display systems. The signal can also be used directly by some larger

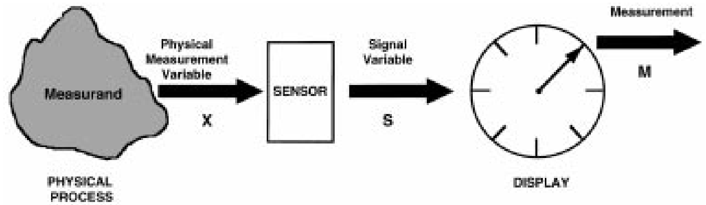

FIGURE 1.1 Simple instrument model.

TABLE 1.1

Common physical variables	Typical signal variables
• Force	• Voltage
• Length	• Displacement
• Temperature	• Current
• Acceleration	• Force
• Velocity	• Pressure
• Pressure	• Light
• Frequency	• Frequency
• Capacity	
• Resistance	
• Time	
• …	

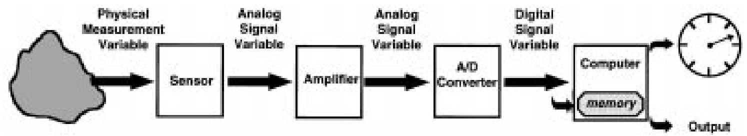

FIGURE 1.2 Instrument model with amplifier, analog to digital converter, and computer output.

system of which the instrument is a part. For example, the output signal of the sensor may be used as the input signal of a closed loop control system.

If the signal output from the sensor is small, it is sometimes necessary to amplify the output shown in Figure 1.2. The amplified output can then be transmitted to the display device or recorded, depending on the particular measurement application. In many cases it is necessary for the instrument to provide a digital signal output so that it can interface with a computer-based data acquisition or communications system. If the sensor does not inherently provide a digital output, then the analog output of the sensor is converted by an analog to digital converter (ADC) as shown in Figure 1.2. The digital signal is typically sent to a computer processor that can display, store, or transmit the data as output to some other system, which will use the measurement.

Passive and Active Sensors

As discussed above, sensors convert physical variables to signal variables. Sensors are often transducers in that they are devices that convert input energy of one form into output energy of another form. Sensors

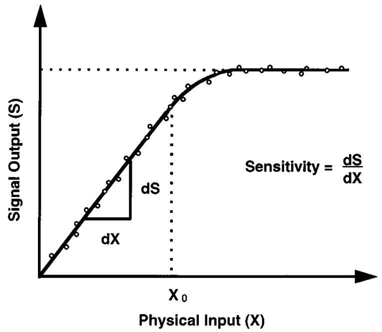

FIGURE 1.3 Calibration curve example.

can be categorized into two broad classes depending on how they interact with the environment they are measuring. *Passive sensors* do not add energy as part of the measurement process but may remove energy in their operation. One example of a passive sensor is a thermocouple, which converts a physical temperature into a voltage signal. In this case, the temperature gradient in the environment generates a thermoelectric voltage that becomes the signal variable. Another passive transducer is a pressure gage where the pressure being measured exerts a force on a mechanical system (diaphragm, aneroid or Borden pressure gage) that converts the pressure force into a displacement, which can be used as a signal variable. For example, the displacement of the diaphragm can be transmitted through a mechanical gearing system to the displacement of an indicating needle on the display of the gage.

Active sensors add energy to the measurement environment as part of the measurement process. An example of an active sensor is a radar or sonar system, where the distance to some object is measured by actively sending out a radio (radar) or acoustic (sonar) wave to reflect off of some object and measure its range from the sensor.

Calibration

The relationship between the physical measurement variable input and the signal variable (output) for a specific sensor is known as the *calibration* of the sensor. Typically, a sensor (or an entire instrument system) is calibrated by providing a known physical input to the system and recording the output. The data are plotted on a calibration curve such as the example shown in Figure 1.3. In this example, the sensor has a linear response for values of the physical input less than X_0. The *sensitivity* of the device is determined by the slope of the calibration curve. In this example, for values of the physical input greater than X_0, the calibration curve becomes less sensitive until it reaches a limiting value of the output signal. This behavior is referred to as *saturation,* and the sensor cannot be used for measurements greater than its saturation value. In some cases, the sensor will not respond to very small values of the physical input variable. The difference between the smallest and largest physical inputs that can reliably be measured by an instrument determines the *dynamic range* of the device.

Modifying and Interfering Inputs

In some cases, the sensor output will be influenced by physical variables other than the intended measurand. In Figure 1.4, ***X*** is the intended measurand, ***Y*** is an *interfering input*, and ***Z*** is a *modifying input.* The interfering input ***Y*** causes the sensor to respond in the same manner as the linear superposition of ***Y*** and the intended measurand ***X***. The measured signal output is therefore a combination of ***X*** and ***Y***,

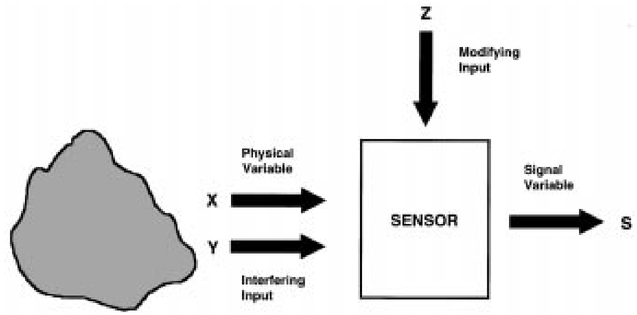

FIGURE 1.4 Interfering inputs.

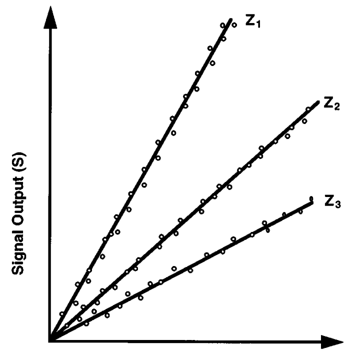

FIGURE 1.5 Illustration of the effect of a modifying input on a calibration curve.

with ***Y*** interfering with the intended measurand ***X***. An example of an interfering input would be a structural vibration within a force measurement system.

Modifying inputs changes the behavior of the sensor or measurement system, thereby modifying the input/output relationship and calibration of the device. This is shown schematically in Figure 1.5. For various values of ***Z*** in Figure 1.5, the slope of the calibration curve changes. Consequently, changing ***Z*** will result in a change of the apparent measurement even if the physical input variable ***X*** remains constant. A common example of a modifying input is temperature; it is for this reason that many devices are calibrated at specified temperatures.

Accuracy and Error

The *accuracy* of an instrument is defined as the difference between the *true value* of the measurand and the *measured value* indicated by the instrument. Typically, the true value is defined in reference to some absolute or agreed upon standard. For any particular measurement there will be some error due to

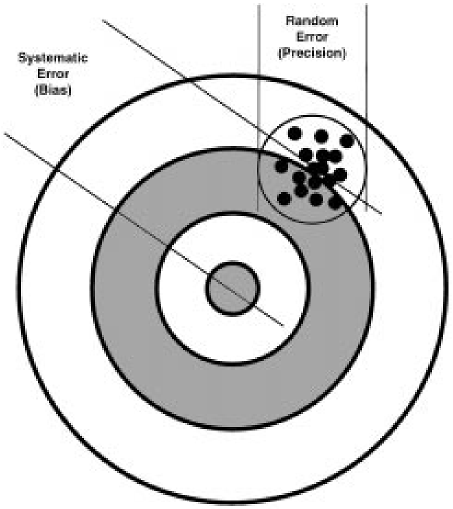

FIGURE 1.6 Target analogy of measurement accuracy.

systematic (*bias*) and *random* (*noise*) error sources. The combination of systematic and random error can be visualized by considering the analogy of the target shown in Figure 1.6. The total error in each shot results from both systematic and random errors. The systematic (bias) error results in the grouping of shots being offset from the bulls eye (presumably a misalignment of the gunsight or wind). The size of the grouping is determined by random error sources and is a measure of the *precision* of the shooting.

Systematic Error Sources (Bias)

There are a variety of factors that can result in systematic measurement errors. One class of cause factors are those that change the input–output response of a sensor resulting in miscalibration. The modifying inputs and interfering inputs discussed above can result in sensor miscalibration. For example, if temperature is a modifying input, using the sensor at a temperature other than the calibrated temperature will result in a systematic error. In many cases, if the systematic error source is known, it can be corrected for by the use of *compensation methods*.

There are other factors that can also cause a change in sensor calibration resulting in systematic errors. In some sensors, aging of the components will change the sensor response and hence the calibration. Damage or abuse of the sensor can also change the calibration. In order to prevent these systematic errors, sensors should be periodically recalibrated.

Systematic errors can also be introduced if the measurement process itself changes the intended measurand. This issue, defined as *invasiveness,* is a key concern in many measurement problems. Interaction between measurement and measurement device is always present; however, in many cases, it can be reduced to an insignificant level. For example, in electronic systems, the energy drain of a measuring device can be made negligible by making the input impedance very high. An extreme example of invasiveness would be to use a large warm thermometer to measure the temperature of a small volume of cold fluid. Heat would be transferred from the thermometer and would warm the fluid, resulting in an inaccurate measurement.

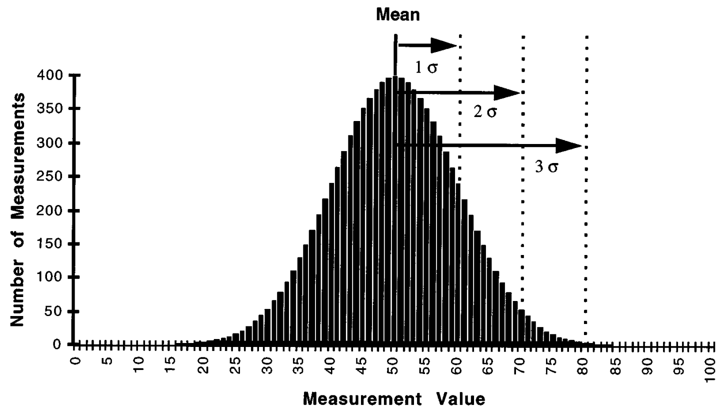

FIGURE 1.7 Example of a Gaussian distribution.

Systematic errors can also be introduced in the signal path of the measurement process shown in Figure 1.3. If the signal is modified in some way, the indicated measurement will be different from the sensed value. In physical signal paths such as mechanical systems that transmit force or displacement, friction can modify the propagation of the signal. In electrical circuits, resistance or attenuation can also modify the signal, resulting in a systematic error.

Finally, systematic errors or bias can be introduced by human observers when reading the measurement. A common example of observer bias error is *parallax error.* This is the error that results when an observer reads a dial from a non-normal angle. Because the indicating needle is above the dial face, the apparent reading will be shifted from the correct value.

Random Error Sources (Noise)

If systematic errors can be removed from a measurement, some error will remain due to the random error sources that define the precision of the measurement. Random error is sometimes referred to as *noise,* which is defined as a signal that carries no useful information. If a measurement with true random error is repeated a large number of times, it will exhibit a *Gaussian distribution,* as demonstrated in the example in Figure 1.7 by plotting the number of times values within specific ranges are measured. The Gaussian distribution is centered on the true value (presuming no systematic errors), so the mean or average of all the measurements will yield a good estimate of the true value.

The precision of the measurement is normally quantified by the standard deviation (σ) that indicates the width of the Gaussian distribution. Given a large number of measurements, a total of 68% of the measurements will fall within $\pm 1\sigma$ of the mean; 95% will fall within $\pm 2\sigma$; and 99.7% will fall within $\pm 3\sigma$. The smaller the standard deviation, the more precise the measurement. For many applications, it is common to refer to the 2σ value when reporting the precision of a measurement. However, for some applications such as navigation, it is common to report the 3σ value, which defines the limit of likely uncertainty in the measurement.

There are a variety of sources of randomness that can degrade the precision of the measurement — starting with the repeatability of the measurand itself. For example, if the height of a rough surface is to be measured, the measured value will depend on the exact location at which the measurement is taken. Repeated measurements will reflect the randomness of the surface roughness.

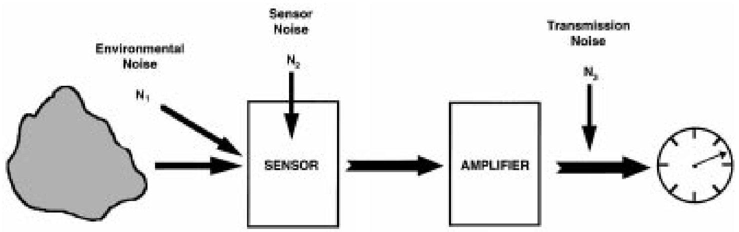

FIGURE 1.8 Instrument model with noise sources.

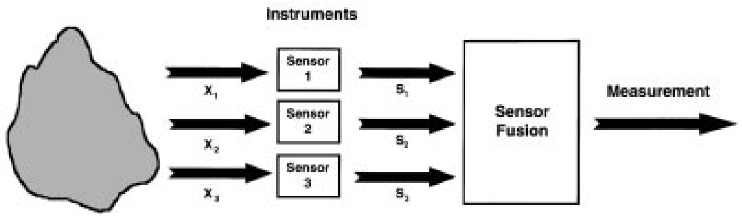

FIGURE 1.9 Example of sensor fusion.

Random error generating noise can also be introduced at each stage in the measurement process, as shown schematically in Figure 1.8. Random interfering inputs will result in noise from the measurement environment N_1 that are introduced before the sensor, as shown in the figure. An example would be background noise received by a microphone. Sensor noise N_2 can also be introduced within the sensor. An example of this would be thermal noise within a sensitive transducer, such as an infrared sensor. Random motion of electrons, due to temperature, appear as voltage signals, which are apparently due to the high sensitivity of the device. For very sensitive measurements with transducers of this type (e.g., infrared detectors), it is common to cool the detector to minimize this noise source.

Noise N_3 can also be introduced in the transmission path between the transducer and the amplifier. A common example of transmission noise in the U.S. is 60 Hz interference from the electric power grid that is introduced if the transmission path is not well grounded, or if an inadvertent electric grand loop causes the wiring to act as an antenna.

It is important to note that the noise will be amplified along with the signal as it passes through the amplifier in Figure 1.8. As a consequence, the figure of merit when analyzing noise is not the level of the combined noise sources, but the *signal to noise ratio* (*SNR*), defined as the ratio of the signal power to the power in the combined noise sources. It is common to report SNR in decibel units.

The SNR is ideally much greater than 1 (0 dB). However, it is sometimes possible to interpret a signal that is lower than the noise level if some identifying characteristics of that signal are known and sufficient signal processing power is available. The human ability to hear a voice in a loud noise environment is an example of this signal processing capability.

Sensor Fusion

The process of *sensor fusion* is modeled in Figure 1.9. In this case, two or more sensors are used to observe the environment and their output signals are combined in some manner (typically in a processor) to

provide a single enhanced measurement. This process frequently allows measurement of phenomena that would otherwise be unobservable. One simple example is thermal compensation of a transducer where a measurement of temperature is made and used to correct the transducer output for modifying effects of temperature on the transducer calibration. Other more sophisticated sensor fusion applications range to image synthesis where radar, optical, and infrared images can be combined into a single enhanced image.

Estimation

With the use of computational power, it is often possible to improve the accuracy of a poor quality measurement through the use of *estimation techniques.* These methods range from simple averaging or low-pass filtering to cancel out random fluctuating errors to more sophisticated techniques such as Wiener or Kalman filtering and model-based estimation techniques. The increasing capability and lowering cost of computation makes it increasingly attractive to use lower performance sensors with more sophisticated estimation techniques in many applications.

Operational Modes of Instrumentation

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2.1 Null Instrument

The null method is one possible mode of operation for a measuring instrument. A **null instrument** uses the null method for measurement. In this method, the instrument exerts an influence on the measured system so as to oppose the effect of the measurand. The influence and the measurand are balanced until they are equal but opposite in value, yielding a null measurement. Typically, this is accomplished by some type of feedback operation that allows the comparison of the measurand against a known standard value. Key features of a null instrument include: an iterative balancing operation using some type of comparator, either a manual or automatic feedback used to achieve balance, and a null deflection at parity.

A null instrument offers certain intrinsic advantages over other modes of operation (e.g., see deflection instruments). By balancing the unknown input against a known standard input, the null method minimizes interaction between the measuring system and the measurand. As each input comes from a separate source, the significance of any measuring influence on the measurand by the measurement process is reduced. In effect, the measured system sees a very high input impedance, thereby minimizing loading errors. This is particularly effective when the measurand is a very small value. Hence, the null operation can achieve a high accuracy for small input values and a low loading error. In practice, the null instrument will not achieve perfect parity due to the usable resolution of the balance and detection methods, but this is limited only by the state of the art of the circuit or scheme being employed.

A disadvantage of null instruments is that an iterative balancing operation requires more time to execute than simply measuring sensor input. Thus, this method might not offer the fastest measurement possible when high-speed measurements are required. However, the user should weigh achievable accuracy against needed speed of measurement when considering operational modes. Further, the design of the comparator and balance loop can become involved such that highly accurate devices are generally not the lowest cost measuring alternative.

An equal arm balance scale is a good mechanical example of a manual balance-feedback null instrument, as shown in Figure 2.1. This scale compares the unknown weight of an object on one side against a set of standard or known weights. Known values of weight are iteratively added to one side to exert an influence to oppose the effect of the unknown weight on the opposite side. Until parity, a high or low value is noted by the indicator providing the feedback logic to the operator for adding or removing

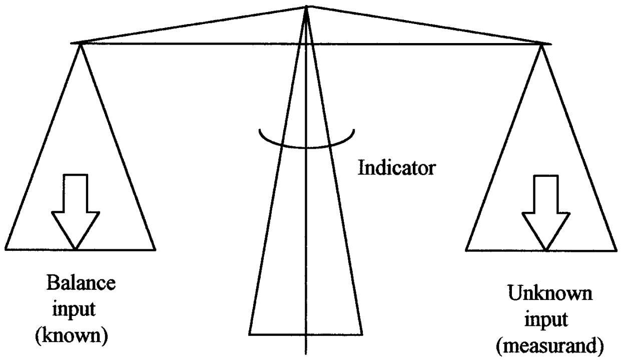

FIGURE 2.1 The measurand and the known quantities balance one another in a null instrument.

weights in a balancing iteration. At true parity, the scale indicator is null; that is, it indicates a zero deflection. Then, the unknown input or measurand is deduced to have a value equal to the balance input, the amount of known weights used to balance the scale. Factors influencing the overall measurement accuracy include the accuracy of the standard weights used and resolution of the output indicator, and the friction at the fulcrum. Null instruments exist for measurement of most variables. Other common examples include bridge circuits, often employed for highly accurate resistance measurements and found in load cells, temperature-compensated transducers, and voltage balancing potentiometers used for highly accurate low-voltage measurements.

Within the null instrument, the iteration and feedback mechanism is a loop that can be controlled either manually or automatically. Essential to the null instrument are two inputs: the measurand and the balance input. The null instrument includes a differential comparator, which compares and computes the difference between these two inputs. This is illustrated in Figure 2.2. A nonzero output from the comparator provides the error signal and drives the logic for the feedback correction. Repeated corrections provide for an iteration toward eventual parity between the inputs and results in the null condition where the measurand is exactly opposed by the balance input. At parity, the error signal is driven to zero by the opposed influence of the balance input and the indicated deflection is at null, thus lending the name to the method. It is the magnitude of the balance input that drives the output reading in terms of the measurand.

2.2 Deflection Instrument

The deflection method is one possible mode of operation for a measuring instrument. A **deflection instrument** uses the deflection method for measurement. A deflection instrument is influenced by the measurand so as to bring about a proportional response within the instrument. This response is an output reading that is a deflection or a deviation from the initial condition of the instrument. In a typical form, the measurand acts directly on a prime element or primary circuit so as to convert its information into a detectable form. The name is derived from a common form of instrument where there is a physical deflection of a prime element that is linked to an output scale, such as a pointer or other type of readout, which deflects to indicate the measured value. The magnitude of the deflection of the prime element brings about a deflection in the output scale that is designed to be proportional in magnitude to the value of the measurand.

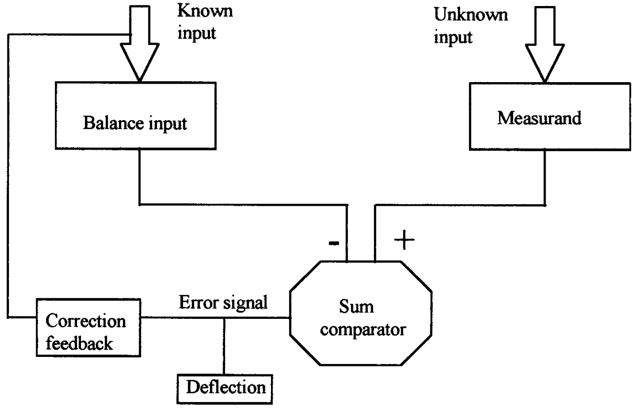

FIGURE 2.2 A null instrument requires input from two sources for comparison.

Deflection instruments are the most common of measuring instruments. The relationship between the measurand and the prime element or measuring circuit can be a direct one, with no balancing mechanism or comparator circuits used. The proportional response can be manipulated through signal conditioning methods between the prime element and the output scale so that the output reading is a direct indication of the measurand. Effective designs can achieve a high accuracy, yet sufficient accuracy for less demanding uses can be achieved at moderate costs.

An attractive feature of the deflection instrument is that it can be designed for either static or dynamic measurements or both. An advantage to deflection design for dynamic measurements is in the high dynamic response that can be achieved. A disadvantage of deflection instruments is that by deriving its energy from the measurand, the act of measurement will influence the measurand and change the value of the variable being measured. This change is called a loading error. Hence, the user must ensure that the resulting error is acceptable. This usually involves a careful look at the instrument input impedance for the intended measurement.

A spring scale is a good, simple example of a deflection instrument. As shown in Figure 2.3, the input weight or measurand acts on a plate-spring. The plate-spring serves as a prime element. The original position of the spring is influenced by the applied weight and responds with a translational displacement, a deflection x. The final value of this deflection is a position that is at equilibrium between the downward force of the weight, W, and the upward restoring force of the spring, kx. That is, the input force is balanced against the restoring force. A mechanical coupler is connected directly or by linkage to a pointer. The pointer position is mapped out on a corresponding scale that serves as the readout scale. For example, at equilibrium $W = kx$ or by measuring the deflection of the pointer the weight is deduced by $x = W/k$.

The flow diagram logic for a deflection instrument is rather linear, as shown if Figure 2.4. The input signal is sensed by the prime element or primary circuit and thereby deflected from its initial setting. The deflection signal is transmitted to signal conditioners that act to condition the signal into a desired form. Examples of signal conditioning are to multiply the deflection signal by some scaler magnitude, such as in amplification or filtering, or to transform the signal by some arithmetic function. The conditioned signal is then transferred to the output scale, which provides the indicated value corresponding to the measurand value.

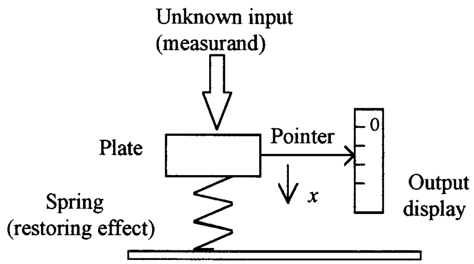

FIGURE 2.3 A deflection instrument requires input from only one source, but may introduce a loading error.

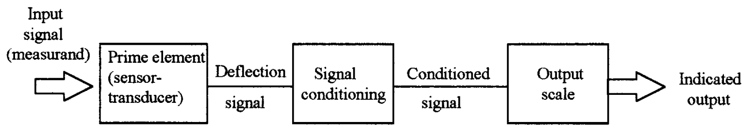

FIGURE 2.4 The logic flow chart for a deflection instrument is straightforward.

2.3 Analog and Digital Sensors

Analog sensors provide a signal that is continuous in both its magnitude and its temporal (time) or spatial (space) content. The defining word for analog is "continuous." If a sensor provides a continuous output signal that is directly proportional to the input signal, then it is analog.

Most physical variables, such as current, temperature, displacement, acceleration, speed, pressure, light intensity, and strain, tend to be continuous in nature and are readily measured by an analog sensor and represented by an analog signal. For example, the temperature within a room can take on any value within its range, will vary in a continuous manner in between any two points in the room, and may vary continuously with time at any position within the room. An analog sensor, such as a bulb thermometer or a thermocouple, will continuously respond to such temperature changes. Such a continuous signal is shown in Figure 2.5, where the signal magnitude is analogous to the measured variable (temperature) and the signal is continuous in both magnitude and time.

Digital sensors provide a signal that is a direct digital representation of the measurand. Digital sensors are basically binary ("on" or "off") devices. Essentially, a digital signal exists at only discrete values of

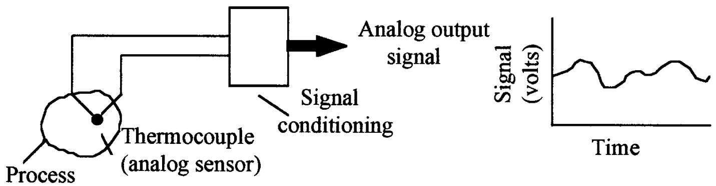

FIGURE 2.5 A thermocouple provides an analog signal for processing.

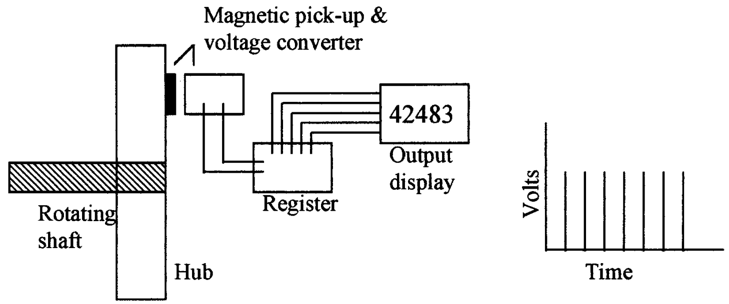

FIGURE 2.6 A rotating shaft with a revolution counter produces a digital signal.

time (or space). And within that discrete period, the signal can represent only a discrete number of magnitude values. A common variation is the **discrete sampled signal** representation, which represents a sensor output in a form that is discrete both in time or space and in magnitude.

Digital sensors use some variation of a binary numbering system to represent and transmit the signal information in digital form. A binary numbering system is a number system using the base 2. The simplest binary signal is a single bit that has only one of two possible values, a 1 or a 0. Bits are like electrical "on-off" switches and are used to convey logical and numerical information. With appropriate input, the value of the bit transmitted is reset corresponding to the behavior of the measured variable. A digital sensor that transmits information one bit at a time uses serial transmission. By combining bits or transmitting bits in groups, it is also possible to define logical commands or integer numbers beyond a 0 or 1. A digital sensor that transmits bits in groups uses parallel transmission. With any digital device, an M-bit signal can express 2^M different numbers. This also provides the limit for the different values that a digital device can discern. For example, a 2-bit device can express 2^2 or 4 different numbers, 00, 01, 10, and 11, corresponding to the values of 0, 1, 2, and 3, respectively. Thus, the resolution in a magnitude discerned by a digital sensor is inherently limited to 1 part in 2^M.

The concept of a digital sensor is illustrated by the revolution counter in Figure 2.6. Such devices are widely used to sense the revolutions per minute of a rotating shaft. In this example, the sensor is a magnetic pick-up/voltage converter that outputs a pulse with each pass of a magnetic stud mounted to a hub on the rotating shaft. The output from the pick-up normally is "off" but is momentarily turned "on" by the passing stud. This pulse is a voltage spike sent to a digital register whose value is increased by a single count with each spike. The register can send the information to an output device, such as the digital display shown. The output from the sensor can be viewed in terms of voltage spikes with time. The count rate is related to the rotational speed of the shaft. As seen, the signal is discrete in time. A single stud with pick-up will increase the count by one for each full rotation of the shaft. Fractions of a rotation can be resolved by increasing the number of studs on the hub. In this example, the continuous rotation of the shaft is analog but the revolution count is digital. The amplitude of the voltage spike is set to activate the counter and is not related to the shaft rotational speed.

2.4 Analog and Digital Readout Instruments

An **analog readout instrument** provides an output indication that is continuous and directly analogous to the behavior of the measurand. Typically, this might be the deflection of a pointer or an ink trace on a graduated scale, or the intensity of a light beam or a sound wave. This indicated deflection may be driven by changes in voltage or current, or by mechanical, magnetic, or optical means, or combinations

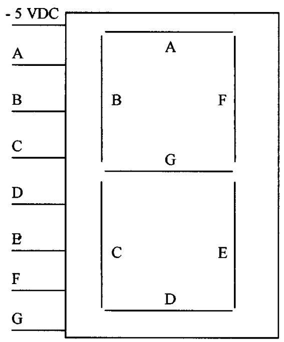

FIGURE 2.7 A seven-segment display chip can display any digit from 0 to 9.

of these. The resolution of an analog readout is defined by the smallest usable increment on its readout scale. The span of the readout is defined by the difference between the minimum and maximum values that it can indicate. Its range specifies the minimum and maximum values that it can indicate.

A **digital readout instrument** provides an output indication that is discrete. The value of the digital output is directly related to the value of the measurand. The digital readout is typically in the form of a numerical value that is either a fixed number or a number that is updated periodically. One means of displaying a digital number is the seven-segment digital display chip, shown in Figure 2.7, whose output can be updated by altering the grounding inputs A through G. The resolution of a digital readout is given by its least count, the equivalent amount of the smallest change resolved by the least significant digit in the readout. The span and range are defined as for analog instruments.

Many digital devices combine features of an analog sensor with a digital readout or, in general, convert an analog signal to a discrete signal, which is indicated through a digital output. In such situations, an analog to digital converter (ADC) is required. This hybrid device has its analog side specified in terms of its full-scale analog range, E_{FSR}, which defines the analog voltage span over which the device will operate. The digital side is specified in terms of the bit size of its register. An M-bit device will output an M-bit binary number. The resolution of such a device is given by $E_{FSR}/2^M$.

2.5 Input Impedance

In the ideal sense, the very act of measurement should not alter the value of the measured signal. Any such alteration is a **loading error**. Loading errors can occur at any junction along the signal chain but can be minimized by impedance matching of the source with the measuring instrument. The measuring instrument input impedance controls the energy that is drawn from the source, or measured system, by a measuring instrument. The power loss through the measuring instrument is estimated by $P = E^2/Z_2$ where Z_2 is the input impedance of the measuring instrument, and E is the source voltage potential being measured. Thus, to minimize the power loss, the input impedance should be large.

This same logic holds for the two instruments in a signal chain as the subsequent instrument draws energy from the previous instrument in the chain. As a general example, consider the situation in Figure 2.8 in which the output signal from one instrument provides the input signal to a subsequent device in a signal chain. The open circuit potential, E_1, is present at the output terminal of source device 1 having output impedance, Z_1. Device 2 has an **input impedance** Z_2 at its input terminals. Connecting

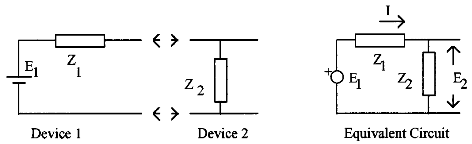

FIGURE 2.8 An equivalent circuit is formed by applying a measuring instrument to the output terminals of an instrument.

the output terminals of device 1 to the input terminals of device 2 creates the equivalent circuit also shown in Figure 2.7. The potential actually sensed by device 2 will be

$$E_2 = E_1 \frac{1}{1 + Z_1 / Z_2}$$

The difference between the actual potential E_1 at the output terminals of device 1 and the measured potential E_2 is a **loading error** brought on by the input impedance of measuring device 2. It is clear that a high input impedance Z_2 relative to Z_1 minimizes this error. A general rule is for the input impedance to be at least 100 times the source impedance to reduce the loading error to 1%.

In general, null instruments and null methods will minimize loading errors. They provide the equivalent of a very high input impedance to the measurement, minimizing energy drain from the measured system. Deflection instruments and deflection measuring techniques will derive energy from the process being measured and therefore require attention to proper selection of input impedance.

Defining Terms

Analog sensor: Sensors that output a signal that is continuous in both magnitude and time (or space).
Deflection instrument: A measuring device whose output deflects proportional to the magnitude of the measurand.
Digital sensor: Sensors that output a signal that is discrete (noncontinuous) in time and/or magnitude.
Input impedance: The impedance measured across the input terminals of a device.
Loading error: That difference between the measurand and the measuring system output attributed to the act of measuring the measurand.
Measurand: A physical quantity, property, or condition being measured. Often, it is referred to as a measured value.
Null instrument: A measuring device that balances the measurand against a known value, thus achieving a null condition. A null instrument minimizes measurement loading errors.
Readout: This is the display of a measuring system.
Resolution: This is the least count or smallest detectable change in measurand capable.
Sensor: The portion of a measurement system that responds directly to the physical variable being measured.

Further Information

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Static and Dynamic Characteristics of Instrumentation

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Before we can begin to develop an understanding of the static and time changing characteristics of measurements, it is necessary to build a framework for understanding the process involved, setting down the main words used to describe concepts as we progress.

Measurement is the process by which relevant information about a system of interest is interpreted using the human thinking ability to define what is believed to be the new knowledge gained. This information may be obtained for purposes of controlling the behavior of the system (as in engineering applications) or for learning more about it (as in scientific investigations).

The basic entity needed to develop the knowledge is called *data*, and it is obtained with physical assemblies known as sensors that are used to observe or sense system variables. The terms *information* and *knowledge* tend to be used interchangeably to describe the entity resulting after data from one or more sensors have been processed to give more meaningful understanding. The individual variables being sensed are called *measurands*.

The most obvious way to make observations is to use the human senses of seeing, feeling, and hearing. This is often quite adequate or may be the only means possible. In many cases, however, sensors are used that have been devised by man to enhance or replace our natural sensors. The number and variety of sensors is very large indeed. Examples of man-made sensors are those used to measure temperature, pressure, or length. The process of sensing is often called *transduction*, being made with transducers. These man-made sensor assemblies, when coupled with the means to process the data into knowledge, are generally known as (measuring) instrumentation.

The degree of perfection of a measurement can only be determined if the goal of the measurement can be defined without error. Furthermore, instrumentation cannot be made to operate perfectly. Because of these two reasons alone, measuring instrumentation cannot give ideal sensing performance and it must be selected to suit the allowable error in a given situation.

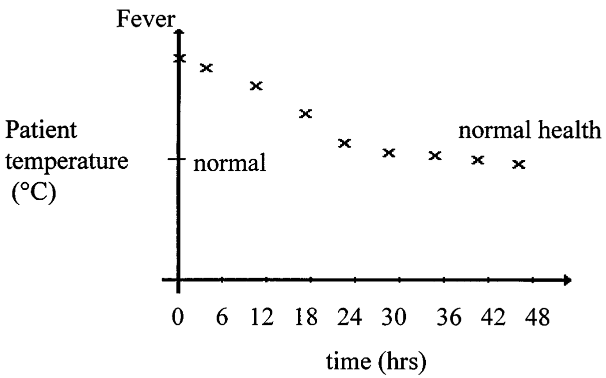

FIGURE 3.1 A patient's temperature chart shows changes taking place over time.

Measurement is a process of mapping actually occurring variables into equivalent values. Deviations from perfect measurement mappings are called *errors*: what we get as the result of measurement is not exactly what is being measured. A certain amount of error is allowable provided it is below the level of uncertainty we can accept in a given situation. As an example, consider two different needs to measure the measurand, time. The uncertainty to which we must measure it for daily purposes of attending a meeting is around a 1 min in 24 h. In orbiting satellite control, the time uncertainty needed must be as small as milliseconds in years. Instrumentation used for the former case costs a few dollars and is the watch we wear; the latter instrumentation costs thousands of dollars and is the size of a suitcase.

We often record measurand values as though they are constant entities, but they usually change in value as time passes. These "dynamic" variations will occur either as changes in the measurand itself or where the measuring instrumentation takes time to follow the changes in the measurand — in which case it may introduce unacceptable error.

For example, when a fever thermometer is used to measure a person's body temperature, we are looking to see if the person is at the normally expected value and, if it is not, to then look for changes over time as an indicator of his or her health. Figure 3.1 shows a chart of a patient's temperature. Obviously, if the thermometer gives errors in its use, wrong conclusions could be drawn. It could be in error due to incorrect calibration of the thermometer or because no allowance for the dynamic response of the thermometer itself was made.

Instrumentation, therefore, will only give adequately correct information if we understand the static and dynamic characteristics of both the measurand and the instrumentation. This, in turn, allows us to then decide if the error arising is small enough to accept.

As an example, consider the electronic signal amplifier in a sound system. It will be commonly quoted as having an amplification constant after feedback if applied to the basic amplifier of, say, 10. The actual amplification value is dependent on the frequency of the input signal, usually falling off as the frequency increases. The frequency response of the basic amplifier, before it is configured with feedback that markedly alters the response and lowers the amplification to get a stable operation, is shown as a graph of amplification gain versus input frequency. An example of the open loop gain of the basic amplifier is given in Figure 3.2. This lack of uniform gain over the frequency range results in error — the sound output is not a true enough representation of the input.

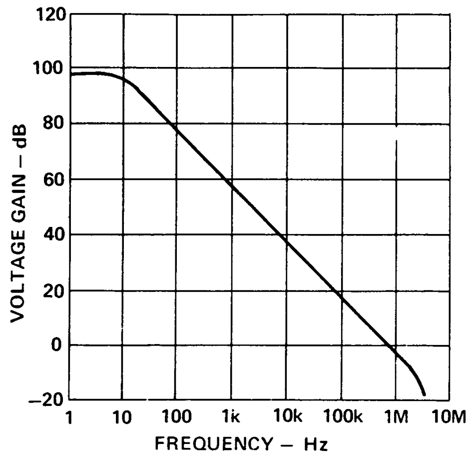

FIGURE 3.2 This graph shows how the amplification of an amplifier changes with input frequency.

Before we can delve more deeply into the static and dynamic characteristics of instrumentation, it is necessary to understand the difference in meaning between several basic terms used to describe the results of a measurement activity.

The correct terms to use are set down in documents called *standards*. Several standardized metrology terminologies exist but they are not consistent. It will be found that books on instrumentation and statements of instrument performance often use terms in different ways. Users of measurement information need to be constantly diligent in making sure that the statements made are interpreted correctly.

The three companion concepts about a measurement that need to be well understood are its *discrimination*, its *precision*, and its *accuracy*. These are too often used interchangeably — which is quite wrong to do because they cover quite different concepts, as will now be explained.

When making a measurement, the smallest increment that can be discerned is called the *discrimination*. (Although now officially declared as wrong to use, the term *resolution* still finds its way into books and reports as meaning discrimination.) The discrimination of a measurement is important to know because it tells if the sensing process is able to sense fine enough changes of the measurand.

Even if the discrimination is satisfactory, the value obtained from a repeated measurement will rarely give exactly the same value each time the same measurement is made under conditions of constant value of measurand. This is because errors arise in real systems. The spread of values obtained indicates the precision of the set of the measurements. The word *precision* is not a word describing a quality of the measurement and is incorrectly used as such. Two terms that should be used here are: *repeatability*, which describes the variation for a set of measurements made in a very short period; and the *reproducibility*, which is the same concept but now used for measurements made over a long period. As these terms describe the outcome of a set of values, there is need to be able to quote a single value to describe the overall result of the set. This is done using statistical methods that provide for calculation of the "mean value" of the set and the associated spread of values, called its *variance*.

The *accuracy* of a measurement is covered in more depth elsewhere so only an introduction to it is required here. Accuracy is the closeness of a measurement to the value defined to be the true value. This

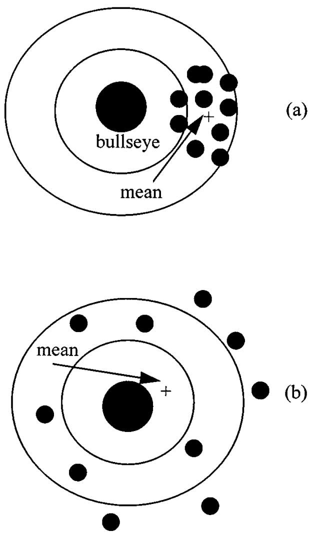

FIGURE 3.3 Two sets of arrow shots fired into a target allow understanding of the measurement concepts of discrimination, precision, and accuracy. (a) The target used for shooting arrows allows investigation of the terms used to describe the measurement result. (b) A different set of placements.

concept will become clearer when the following illustrative example is studied for it brings together the three terms into a single perspective of a typical measurement.

Consider then the situation of scoring an archer shooting arrows into a target as shown in Figure 3.3(*a*). The target has a central point — the bulls-eye. The objective for a perfect result is to get all arrows into the bulls-eye. The rings around the bulls-eye allow us to set up numeric measures of less-perfect shooting performance.

Discrimination is the distance at which we can just distinguish (i.e., discriminate) the placement of one arrow from another when they are very close. For an arrow, it is the thickness of the hole that decides the discrimination. Two close-by positions of the two arrows in Figure 3.3(*a*) cannot be separated easily. Use of thinner arrows would allow finer detail to be decided.

Repeatability is determined by measuring the spread of values of a set of arrows fired into the target over a short period. The smaller the spread, the more precise is the shooter. The shooter in Figure 3.3(*a*) is more precise than the shooter in Figure 3.3(*b*).

If the shooter returned to shoot each day over a long period, the results may not be the same each time for a shoot made over a short period. The mean and variance of the values are now called the *reproducibility* of the archer's performance.

Accuracy remains to be explained. This number describes how well the mean (the average) value of the shots sits with respect to the bulls-eye position. The set in Figure 3.3(*b*) is more accurate than the set in Figure 3.3(*a*) because the mean is nearer the bulls-eye (but less precise!).

At first sight, it might seem that the three concepts of discrimination, precision, and accuracy have a strict relationship in that a better measurement is always that with all three aspects made as high as is affordable. This is not so. They need to be set up to suit the needs of the application.

We are now in a position to explore the commonly met terms used to describe aspects of the static and the dynamic performance of measuring instrumentation.

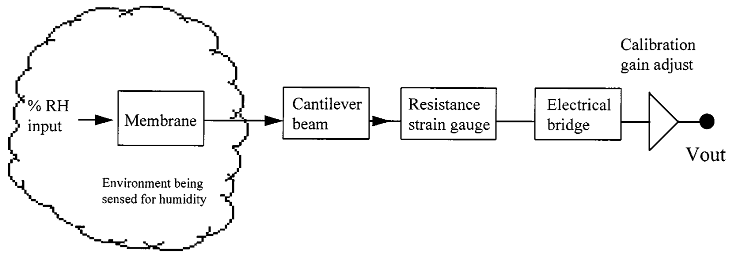

FIGURE 3.4 Instruments are formed from a connection of blocks. Each block can be represented by a conceptual and mathematical model. This example is of one type of humidity sensor.

3.1 Static Characteristics of Instrument Systems

Output/Input Relationship

Instrument systems are usually built up from a serial linkage of distinguishable building blocks. The actual physical assembly may not appear to be so but it can be broken down into a representative diagram of connected blocks. Figure 3.4 shows the block diagram representation of a humidity sensor. The sensor is activated by an input physical parameter and provides an output signal to the next block that processes the signal into a more appropriate state.

A key generic entity is, therefore, the relationship between the input and output of the block. As was pointed out earlier, all signals have a time characteristic, so we must consider the behavior of a block in terms of both the static and dynamic states.

The behavior of the static regime alone and the combined static and dynamic regime can be found through use of an appropriate mathematical model of each block. The mathematical description of system responses is easy to set up and use if the elements all act as linear systems and where addition of signals can be carried out in a linear additive manner. If nonlinearity exists in elements, then it becomes considerably more difficult — perhaps even quite impractical — to provide an easy to follow mathematical explanation. Fortunately, general description of instrument systems responses can be usually be adequately covered using the linear treatment.

The output/input ratio of the whole cascaded chain of blocks 1, 2, 3, etc. is given as:

$$[\text{output/input}]_{\text{total}} = [\text{output/input}]_1 \times [\text{output/input}]_2 \times [\text{output/input}]_3 \ldots$$

The output/input ratio of a block that includes both the static and dynamic characteristics is called the *transfer function* and is given the symbol G.

The equation for G can be written as two parts multiplied together. One expresses the static behavior of the block, that is, the value it has after all transient (time varying) effects have settled to their final state. The other part tells us how that value responds when the block is in its dynamic state. The static part is known as the *transfer characteristic* and is often all that is needed to be known for block description.

The static and dynamic response of the cascade of blocks is simply the multiplication of all individual blocks. As each block has its own part for the static and dynamic behavior, the cascade equations can be rearranged to separate the static from the dynamic parts and then by multiplying the static set and the dynamic set we get the overall response in the static and dynamic states. This is shown by the sequence of Equations 3.1 to 3.4.

$$G_{total} = G_1 \times G_2 \times G_3 \ldots \quad (3.1)$$

$$= [\text{static} \times \text{dynamic}]_1 \times [\text{static} \times \text{dynamic}]_2 \times [\text{static} \times \text{dynamic}]_3 \ldots \quad (3.2)$$

$$= [\text{static}]_1 \times [\text{static}]_2 \times [\text{static}]_3 \ldots \times [\text{dynamic}]_1 \times [\text{dynamic}]_2 \times [\text{dynamic}]_3 \ldots \quad (3.3)$$

$$= [\text{static}]_{total} \times [\text{dynamic}]_{total} \quad (3.4)$$

An example will clarify this. A mercury-in-glass fever thermometer is placed in a patient's mouth. The indication slowly rises along the glass tube to reach the final value, the body temperature of the person. The slow rise seen in the indication is due to the time it takes for the mercury to heat up and expand up the tube. The static *sensitivity* will be expressed as so many scale divisions per degree and is all that is of interest in this application. The dynamic characteristic will be a time varying function that settles to unity after the transient effects have settled. This is merely an annoyance in this application but has to be allowed by waiting long enough before taking a reading. The wrong value will be viewed if taken before the transient has settled.

At this stage, we will now consider only the nature of the static characteristics of a chain; dynamic response is examined later.

If a sensor is the first stage of the chain, the static value of the gain for that stage is called the *sensitivity.* Where a sensor is not at the input, it is called the *amplification factor* or *gain.* It can take a value less than unity where it is then called the *attenuation.*

Sometimes, the instantaneous value of the signal is rapidly changing, yet the measurement aspect part is static. This arises when using ac signals in some forms of instrumentation where the amplitude of the waveform, not its frequency, is of interest. Here, the static value is referred to as its *steady state* transfer characteristic.

Sensitivity may be found from a plot of the input and output signals, wherein it is the slope of the graph. Such a graph, see Figure 3.5, tells much about the static behavior of the block.

The intercept value on the *y*-axis is the *offset* value being the output when the input is set to zero. Offset is not usually a desired situation and is seen as an error quantity. Where it is deliberately set up, it is called the *bias.*

The range on the *x*-axis, from zero to a safe maximum for use, is called the *range* or *span* and is often expressed as the zone between the 0% and 100% points. The ratio of the span that the output will cover

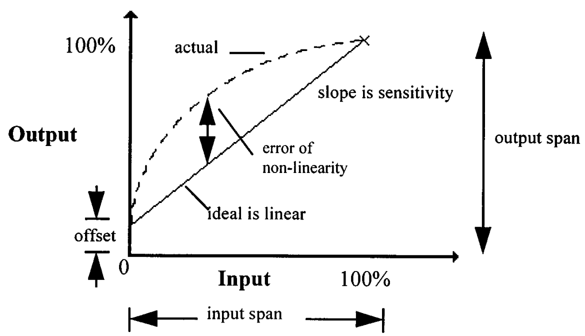

FIGURE 3.5 The graph relating input to output variables for an instrument block shows several distinctive static performance characteristics.

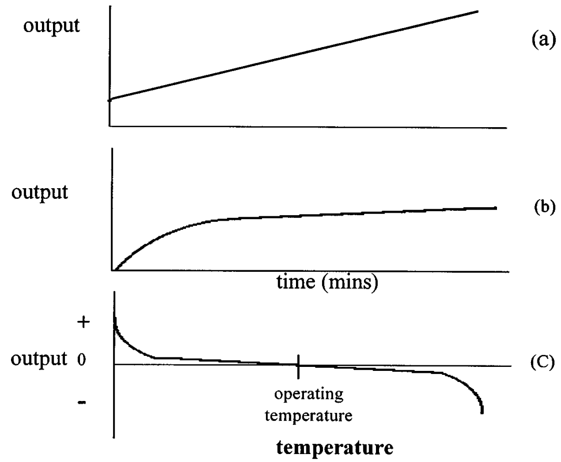

FIGURE 3.6 Drift in the performance of an instrument takes many forms: (*a*) drift over time for a spring balance; (*b*) how an electronic amplifier might settle over time to a final value after power is supplied; (*c*) drift, due to temperature, of an electronic amplifier varies with the actual temperature of operation.

for the related input range is known as the *dynamic range.* This can be a confusing term because it does not describe dynamic time behavior. It is particularly useful when describing the capability of such instruments as flow rate sensors — a simple orifice plate type may only be able to handle dynamic ranges of 3 to 4, whereas the laser Doppler method covers as much as 10^7 variation.

Drift

It is now necessary to consider a major problem of instrument performance called *instrument drift.* This is caused by variations taking place in the parts of the instrumentation over time. Prime sources occur as chemical structural changes and changing mechanical stresses. Drift is a complex phenomenon for which the observed effects are that the sensitivity and offset values vary. It also can alter the accuracy of the instrument differently at the various amplitudes of the signal present.

Detailed description of drift is not at all easy but it is possible to work satisfactorily with simplified values that give the average of a set of observations, this usually being quoted in a conservative manner. The first graph (*a*) in Figure 3.6 shows typical steady drift of a measuring spring component of a weighing balance. Figure 3.6(*b*) shows how an electronic amplifier might settle down after being turned on.

Drift is also caused by variations in environmental parameters such as temperature, pressure, and humidity that operate on the components. These are known as *influence parameters.* An example is the change of the resistance of an electrical resistor, this resistor forming the critical part of an electronic amplifier that sets its gain as its operating temperature changes.

Unfortunately, the observed effects of influence parameter induced drift often are the same as for time varying drift. Appropriate testing of blocks such as electronic amplifiers does allow the two to be separated to some extent. For example, altering only the temperature of the amplifier over a short period will quickly show its temperature dependence.

Drift due to influence parameters is graphed in much the same way as for time drift. Figure 3.6(*c*) shows the drift of an amplifier as temperature varies. Note that it depends significantly on the temperature

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of operation, implying that the best designs are built to operate at temperatures where the effect is minimum.

Careful consideration of the time and influence parameter causes of drift shows they are interrelated and often impossible to separate. Instrument designers are usually able to allow for these effects, but the cost of doing this rises sharply as the error level that can be tolerated is reduced.

Hysteresis and Backlash

Careful observation of the output/input relationship of a block will sometimes reveal different results as the signals vary in direction of the movement. Mechanical systems will often show a small difference in length as the direction of the applied force is reversed. The same effect arises as a magnetic field is reversed in a magnetic material. This characteristic is called *hysteresis.* Figure 3.7 is a generalized plot of the output/input relationship showing that a closed loop occurs. The effect usually gets smaller as the amplitude of successive excursions is reduced, this being one way to tolerate the effect. It is present in most materials. Special materials have been developed that exhibit low hysteresis for their application — transformer iron laminations and clock spring wire being examples.

Where this is caused by a mechanism that gives a sharp change, such as caused by the looseness of a joint in a mechanical joint, it is easy to detect and is known as *backlash.*

Saturation

So far, the discussion has been limited to signal levels that lie within acceptable ranges of amplitude. Real system blocks will sometimes have input signal levels that are larger than allowed. Here, the dominant errors that arise — *saturation* and *crossover distortion* — are investigated.

As mentioned above, the information bearing property of the signal can be carried as the instantaneous value of the signal or be carried as some characteristic of a rapidly varying ac signal. If the signal form is not amplified faithfully, the output will not have the same linearity and characteristics.

The gain of a block will usually fall off with increasing size of signal amplitude. A varying amplitude input signal, such as the steadily rising linear signal shown in Figure 3.8, will be amplified differently according to the gain/amplitude curve of the block. In uncompensated electronic amplifiers, the larger amplitudes are usually less amplified than at the median points.

At very low levels of input signal, two unwanted effects may arise. The first is that small signals are often amplified more than at the median levels. The second error characteristic arises in electronic amplifiers because the semiconductor elements possess a dead-zone in which no output occurs until a small threshold is exceeded. This effect causes crossover distortion in amplifiers.

If the signal is an ac waveform, see Figure 3.9, then the different levels of a cycle of the signal may not all be amplified equally. Figure 3.9(*a*) shows what occurs because the basic electronic amplifying elements are only able to amplify one polarity of signal. The signal is said to be *rectified.* Figure 3.9(*b*) shows the effect when the signal is too large and the top is not amplified. This is called *saturation* or *clipping.* (As with many physical effects, this effect is sometimes deliberately invoked in circuitry, an example being where it is used as a simple means to convert sine-waveform signals into a square waveform.) Crossover distortion is evident in Figure 3.9(*c*) as the signal passes from negative to positive polarity.

Where input signals are small, such as in sensitive sensor use, the form of analysis called *small signal* behavior is needed to reveal distortions. If the signals are comparatively large, as for digital signal considerations, a *large signal* analysis is used. Design difficulties arise when signals cover a wide dynamic range because it is not easy to allow for all of the various effects in a single design.

Bias

Sometimes, the electronic signal processing situation calls for the input signal to be processed at a higher average voltage or current than arises normally. Here a dc value is added to the input signal to raise the level to a higher state as shown in Figure 3.10. A need for this is met where only one polarity of signal

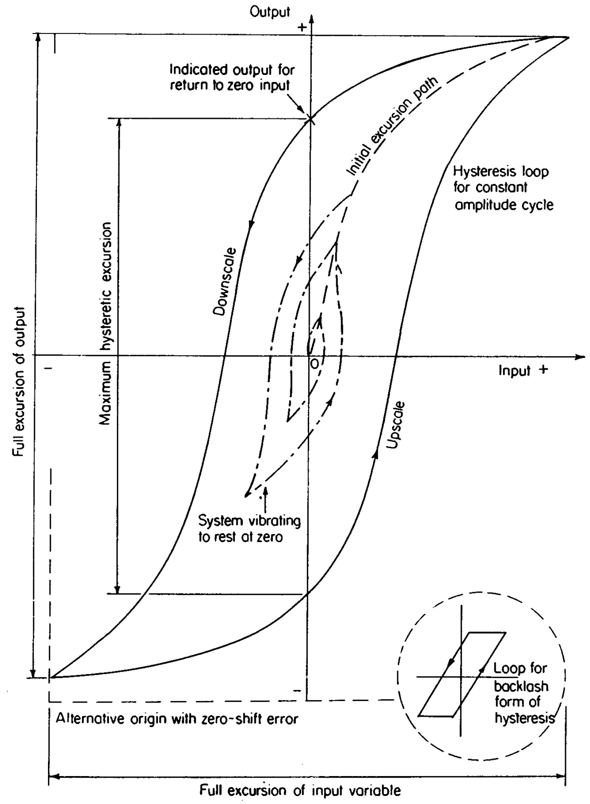

FIGURE 3.7 Generalized graph of output/input relationship where hysteresis is present. (From P. H. Sydenham, *Handbook of Measurement Science,* Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

can be amplified by a single semiconductor element. Raising the level of all of the waveform equally takes all parts into the reasonably linear zone of an amplifier, allowing more faithful replication. If bias were not used here, then the lower half cycle would not be amplified, resulting in only the top half appearing in the output.

Error of Nonlinearity

Ideally, it is often desired that a strictly linear relationship exists between input and output signals in amplifiers. Practical units, however, will always have some degree of nonconformity, which is called the *nonlinearity.* If an instrument block has constant gain for all input signal levels, then the relationship graphing the input against the output will be a straight line; the relationship is then said to be linear.

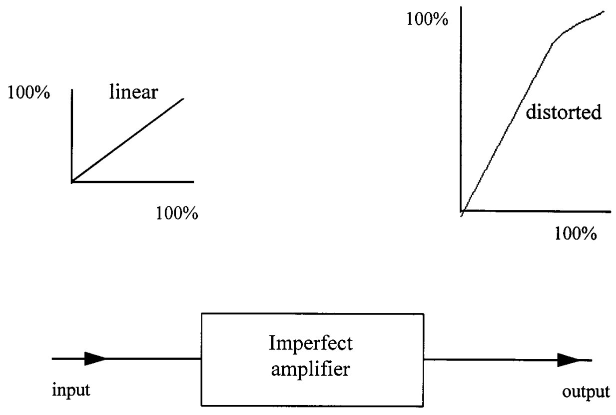

FIGURE 3.8 Nonlinear amplification can give rise to unwanted output distortion.

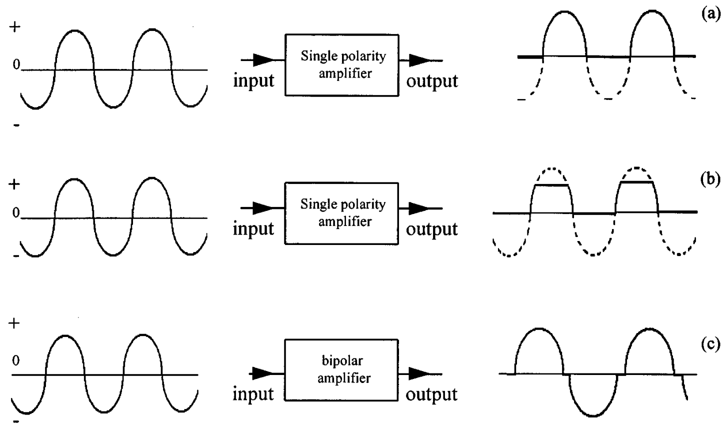

FIGURE 3.9 Blocks can incorrectly alter the shape of waveforms if saturation and crossover effects are not controlled: (*a*) rectification; (*b*) saturation; and (*c*) crossover distortion.

Linearity is the general term used to describe how close the actual response is compared with that ideal line. The correct way to describe the error here is as the *error of nonlinearity*. Note, however, that not all responses are required to be linear; another common one follows a logarithmic relationship.

Detailed description of this error is not easy for that would need a statement of the error values at all points of the plot. Practice has found that a shorthand statement can be made by quoting the maximum departure from the ideal as a ratio formed with the 100% value.

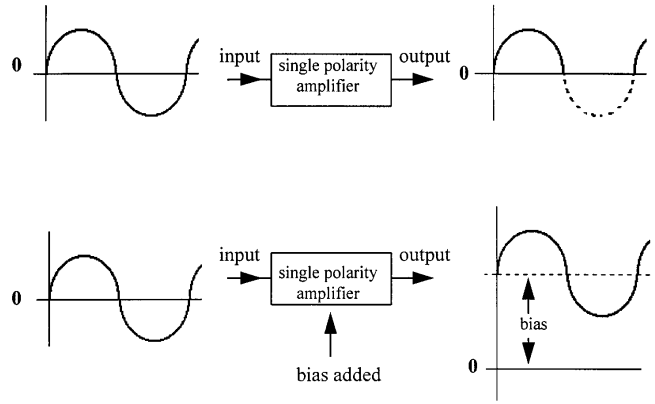

FIGURE 3.10 Bias is where a signal has all of its value raised by an equal amount. Shown here is an ac input waveform biased to be all of positive polarity.

Difficulties arise in expressing error of nonlinearity for there exist many ways to express this error. Figure 3.11 shows the four cases that usually arise. The difference arises in the way in which the ideal (called the "best fit") straight line can be set up. Figure 3.11(*a*) shows the line positioned by the usually calculated statistical averaging method of least squares fit; other forms of line fitting calculation are also used. This will yield the smallest magnitude of error calculation for the various kinds of line fitting but may not be appropriate for how the stage under assessment is used. Other, possibly more reasonable, options exist. Figure 3.11(*b*) constrains the best fit line to pass through the zero point. Figure 3.11(*c*) places the line between the expected 0% and the 100% points. There is still one more option, that where the theoretical line is not necessarily one of the above, yet is the ideal placement, Figure 3.11(*d*).

In practice then, instrument systems linearity can be expressed in several ways. Good certification practice requires that the method used to ascertain the error is stated along with the numerical result, but this is often not done. Note also that the error is the worst case and that part of the response may be much more linear.

The description of instrument performance is not a simple task. To accomplish this fully would require very detailed statements recording the performance at each and every point. That is often too cumbersome, so the instrument industry has developed many short-form statements that provide an adequate guide to the performance. This guide will be seen to be generally a conservative statement.

Many other descriptors exist for the static regime of an instrument. The reader is referred to the many standards documents that exist on instrument terminology; for example, see Reference [3].

3.2 Dynamic Characteristics of Instrument Systems

Dealing with Dynamic States

Measurement outcomes are rarely static over time. They will possess a dynamic component that must be understood for correct interpretation of the results. For example, a trace made on an ink pen chart recorder will be subject to the speed at which the pen can follow the input signal changes.

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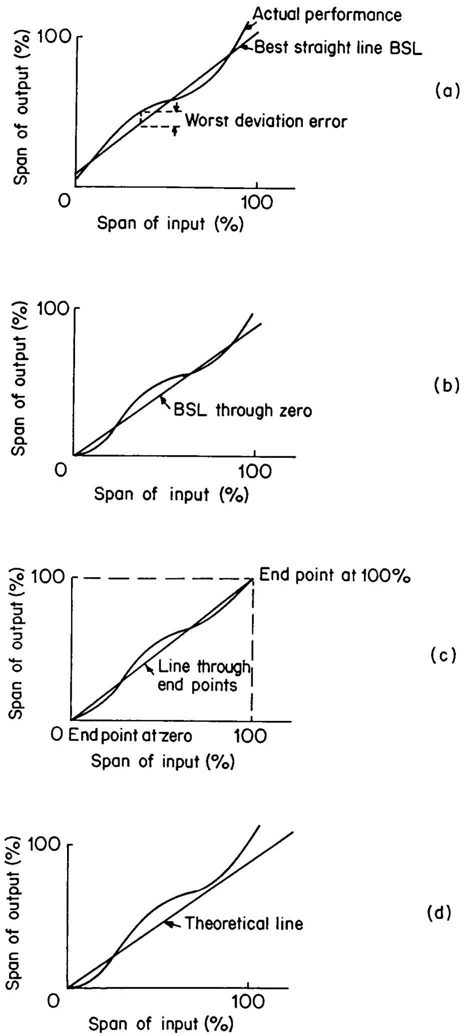

FIGURE 3.11 Error of nonlinearity can be expressed in four different ways: (*a*) best fit line (based on selected method used to decide this); (*b*) best fit line through zero; (*c*) line joining 0% and 100% points; and (*d*) theoretical line. (From P. H. Sydenham, *Handbook of Measurement Science*, Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

To properly appreciate instrumentation design and its use, it is now necessary to develop insight into the most commonly encountered types of dynamic response and to develop the mathematical modeling basis that allows us to make concise statements about responses.

If the transfer relationship for a block follows linear laws of performance, then a generic mathematical method of dynamic description can be used. Unfortunately, simple mathematical methods have not been found that can describe all types of instrument responses in a simplistic and uniform manner. If the behavior is nonlinear, then description with mathematical models becomes very difficult and might be impracticable. The behavior of nonlinear systems can, however, be studied as segments of linear behavior joined end to end. Here, digital computers are effectively used to model systems of any kind provided the user is prepared to spend time setting up an adequate model.

Now the mathematics used to describe linear dynamic systems can be introduced. This gives valuable insight into the expected behavior of instrumentation, and it is usually found that the response can be approximated as linear.

The modeled response at the output of a block G_{result} is obtained by multiplying the mathematical expression for the input signal G_{input} by the transfer function of the block under investigation $G_{response}$, as shown in Equation 3.5.

$$G_{result} = G_{input} \times G_{response} \tag{3.5}$$

To proceed, one needs to understand commonly encountered input functions and the various types of block characteristics. We begin with the former set: the so-called *forcing functions.*

Forcing Functions

Let us first develop an understanding of the various types of input signal used to perform tests. The most commonly used signals are shown in Figure 3.12. These each possess different valuable test features. For example, the sine-wave is the basis of analysis of all complex wave-shapes because they can be formed as a combination of various sine-waves, each having individual responses that add to give all other wave-shapes. The step function has intuitively obvious uses because input transients of this kind are commonly encountered. The ramp test function is used to present a more realistic input for those systems where it is not possible to obtain instantaneous step input changes, such as attempting to move a large mass by a limited size of force. Forcing functions are also chosen because they can be easily described by a simple mathematical expression, thus making mathematical analysis relatively straightforward.

Characteristic Equation Development

The behavior of a block that exhibits linear behavior is mathematically represented in the general form of expression given as Equation 3.6.

$$\ldots\ldots a_2 d^2 y / dt^2 + a_1 dy / dt + a_0 y = x(t) \tag{3.6}$$

Here, the coefficients a_2, a_1, and a_0 are constants dependent on the particular block of interest. The left-hand side of the equation is known as the *characteristic equation.* It is specific to the internal properties of the block and is not altered by the way the block is used.

The specific combination of forcing function input and block characteristic equation collectively decides the combined output response. Connections around the block, such as feedback from the output to the input, can alter the overall behavior significantly: such systems, however, are not dealt with in this section being in the domain of feedback control systems.

Solution of the combined behavior is obtained using Laplace transform methods to obtain the output responses in the time or the complex frequency domain. These mathematical methods might not be familiar to the reader, but this is not a serious difficulty for the cases most encountered in practice are

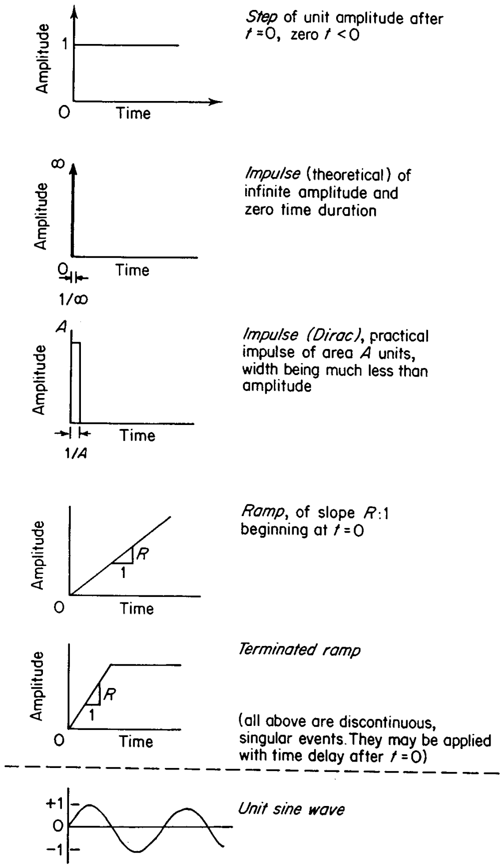

FIGURE 3.12 The dynamic response of a block can be investigated using a range of simple input forcing functions. (From P. H. Sydenham, *Handbook of Measurement Science*, Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

well documented in terms that are easily comprehended, the mathematical process having been performed to yield results that can be used without the same level of mathematical ability. More depth of explanation can be obtained from [1] or any one of the many texts on energy systems analysis. Space here only allows an introduction; this account is linked to [1], Chapter 17, to allow the reader to access a fuller description where needed.

The next step in understanding block behavior is to investigate the nature of Equation 3.6 as the number of derivative terms in the expression increases, Equations 3.7 to 3.10.

Zero order $$a_0 y = x(t) \tag{3.7}$$

First order $$a_1\, dy/dt + a_0 y = x(t) \tag{3.8}$$

Second order $$a_2\, d^2 y/dt^2 + a_1\, dy/dt + a_0 y = x(t) \tag{3.9}$$

*n*th order $$a_n d^n y/dt^n + a_{n-1} d^{n-1} y/dt^{n-1} + \ldots + a_0 y = x(t) \tag{3.10}$$

Note that specific names have been given to each order. The zero-order situation is not usually dealt with in texts because it has no time-dependent term and is thus seen to be trivial. It is an amplifier (or attenuator) of the forcing function with gain of a_0. It has infinite bandwidth without change in the amplification constant.

The highest order usually necessary to consider in first-cut instrument analysis is the second-order class. Higher-order systems do occur in practice and need analysis that is not easily summarized here. They also need deep expertise in their study. Computer-aided tools for systems analysis can be used to study the responses of systems.

Another step is now to rewrite the equations after Laplace transformation into the frequency domain. We then get the set of output/input Equations 3.11 to 3.14.

Zero order $$Y(s)/X(s) = 1 \tag{3.11}$$

First order $$Y(s)/X(s) = 1/(\tau s + 1) \tag{3.12}$$

Second order $$Y(s)/X(s) = 1/(\tau_1 s + 1)(\tau_2 s + 1) \tag{3.13}$$

*n*th order $$Y(s)/X(s) = 1/(\tau_1 s + 1)(\tau_2 s + 1)\ldots(\tau_n s + 1) \tag{3.14}$$

The terms $\tau_1, \ldots, \tau_n$ are called *time constants.* They are key system performance parameters.

Response of the Different Linear Systems Types

Space restrictions do not allow a detailed study of all of the various options. A selection is presented to show how they are analyzed and reported, that leading to how the instrumentation person can use certain standard charts in the study of the characteristics of blocks.

Zero-Order Blocks

To investigate the response of a block, multiply its frequency domain forms of equation for the characteristic equation with that of the chosen forcing function equation.

This is an interesting case because Equation 3.7 shows that the zero-order block has no frequency-dependent term (it has no time derivative term), so the output for all given inputs can only be of the same time form as the input. What can be changed is the amplitude given as the coefficient a_0. A shift in time (phase shift) of the output waveform with the input also does not occur as it can for the higher-order blocks.

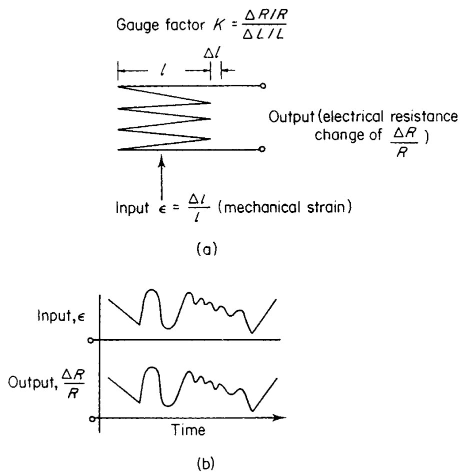

FIGURE 3.13 Input and output responses for a zero-order block: (*a*) strain gage physical and mathematical model; and (*b*) responses. (From P. H. Sydenham, *Handbook of Measurement Science,* Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

This is the response often desired in instruments because it means that the block does not alter the time response. However, this is not always so because, in systems, design blocks are often chosen for their ability to change the time shape of signals in a known manner.

Although somewhat obvious, Figure 3.13, a resistive strain gage, is given to illustrate zero-order behavior.

First-Order Blocks

Here, Equation 3.8 is the relevant characteristic equation. There is a time-dependent term, so analysis is needed to see how this type of block behaves under dynamic conditions. The output response is different for each type of forcing function applied. Space limitations only allow the most commonly encountered cases — the step and the sine-wave input — to be introduced here. It is also only possible here to outline the method of analysis and to give the standardized charts that plot generalized behavior.

The step response of the first-order system is obtained by multiplying Equation 3.12 by the frequency domain equation for a step of amplitude *A*. The result is then transformed back into the time domain using Laplace transforms to yield the expression for the output, $y(t)$

$$y(t) = AK\left(1 - e^{t/\tau}\right) \tag{3.15}$$

where *A* is the amplitude of the step, *K* the static gain of the first-order block, *t* the time in consistent units, and τ the time constant associated with the block itself.

This is a tidy outcome because Equation 3.15 covers the step response for all first-order blocks, thus allowing it to be graphed in normalized manner, as given in Figure 3.14. The shape of the response is always of the same form. This means that the step response of a first-order system can be described as having "a step of *AK* amplitude with the time constant τ."

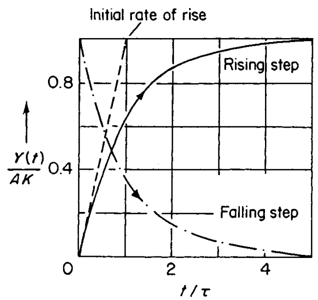

FIGURE 3.14 The step response for all first-order systems is covered by these two normalized graphs. (From P. H. Sydenham, *Handbook of Measurement Science,* Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

If the input is a sine-wave, the output response is quite different; but again, it will be found that there is a general solution for all situations of this kind. As before, the input forcing equation is multiplied by the characteristic equation for the first-order block and Laplace transformation is used to get back to the time domain response. After rearrangement into two parts, this yields:

$$y(t) = \left[AK\tau\omega e^{-t/\tau} \Big/ \left(\tau^2\omega^2 + 1\right)\right] + \left[AK \Big/ \left(\tau^2\omega^2 + 1\right)^{1/2} \cdot \sin\left(\omega t + \phi\right)\right] \tag{3.16}$$

where ω is the signal frequency in angular radians, $\phi = \tan^{-1}(-\omega t)$, *A* the amplitude of the sine-wave input, *K* the gain of the first-order block, *t* the time in consistent units, and τ the time constant associated with the block.

The left side of the right-hand bracketed part is a short-lived, normally ignored, time transient that rapidly decays to zero, leaving a steady-state output that is the parameter of usual interest. Study of the steady-state part is best done by plotting it in a normalized way, as has been done in Figure 3.15.

These plots show that the amplitude of the output is always reduced as the frequency of the input signal rises and that there is always a phase lag action between the input and the output that can range from 0 to 90° but never be more than 90°. The extent of these effects depends on the particular coefficients of the block and input signal. These effects must be well understood when interpreting measurement results because substantial errors can arise with using first-order systems in an instrument chain.

Second-Order Blocks

If the second-order differential term is present, the response of a block is quite different, again responding in quite a spectacular manner with features that can either be wanted or unwanted.

As before, to obtain the output response, the block's characteristic function is multiplied by the chosen forcing function. However, to make the results more meaningful, we first carry out some simple substitution transformations.

The steps begin by transforming the second-order differential Equation 3.6 into its Laplace form to obtain:

$$X(s) = a_2 s^2 Y(s) + a_1 s Y(s) + a_0 Y(s) \tag{3.17}$$

This is then rearranged to yield:

$$G(s) = Y(s) \big/ X(s) = 1/a_0 \cdot 1 \Big/ \left\{\left(a_2/a_0\right)s^2 + \left(a_1/a_0\right)s + 1\right\} \tag{3.18}$$

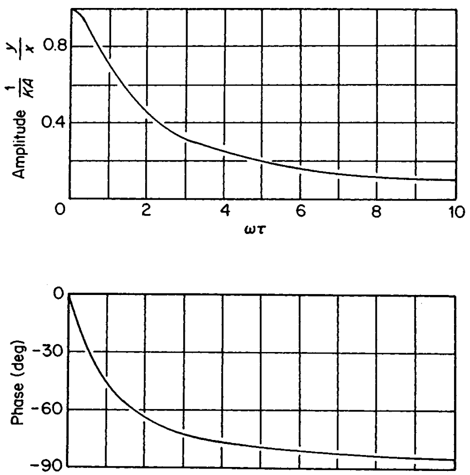

FIGURE 3.15 The amplitude and phase shift of the output of all first-order systems to a sine-wave input is shown by these two normalized curves; (*a*) amplitude and (*b*) phase. (From P. H. Sydenham, *Handbook of Measurement Science,* Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

The coefficients can then be expressed in system performance terms as follows.

Angular natural frequency $$\omega_n = \left(a_0 / a_2\right)^{1/2} \quad (3.19)$$

Damping ratio $$\zeta = a_1 \Big/ 2\left(a_0 \cdot a_2\right)^{1/2} \quad (3.20)$$

Static gain $$K = 1/a_0 \quad (3.21)$$

These three variables have practical relevance, as will be seen when the various responses are plotted.

Using these transformed variables, the characteristic equation can be rewritten in two forms ready for investigation of output behavior to step and sine-wave inputs, as:

$$G(s) = K \Big/ \left\{\left(1/\omega_n^2\right)s^2 + \left(2\zeta/\omega_n\right)s + 1\right\} \quad (3.22)$$

and then as:

$$G(s) = K \Big/ \left(\tau^2 s^2 + 2\zeta\tau s + 1\right) \quad (3.23)$$

We are now ready to consider the behavior of the second-order system to the various forcing inputs.

First consider the step input. After forming the product of the forcing and characteristic functions, the time domain form can be plotted as shown in Figure 3.16.

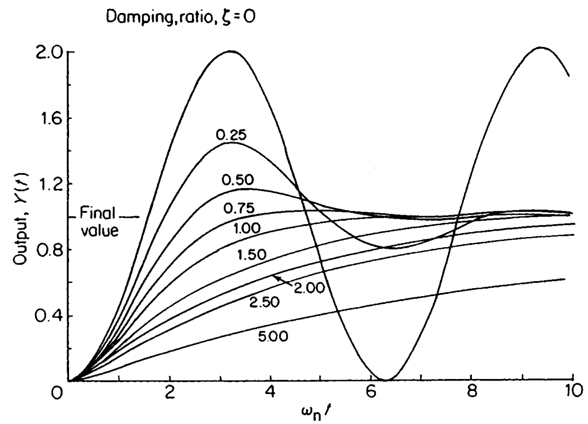

FIGURE 3.16 The response of second-order systems to a step input is seen from this normalized plot. (From P. H. Sydenham, *Handbook of Measurement Science,* Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

This clearly shows that the response is strongly dependent on the damping ratio ζ value. If it is less than unity, it exhibits an oscillatory movement settling down to the final value. If the damping value is greater than unity, the response moves to the final value without oscillation. The often preferred state is to use a damping factor of unity, *critical damping.* The choice of response depends strongly on the applications, for all levels of damping ratio have use in practice, ranging from needing an oscillation that never ceases (zero damping) to the other extreme where a very gradual rate of change is desired.

A similar analysis is used to see how the second-order system responds to the sine-wave input. The two response plots obtained are shown in Figure 3.17: one for the amplitude response, and the other showing how the phase shifts as the frequency changes.

The most unexpected result is seen at the point where the gain rises to infinity for the zero damping state. This is called *resonance* and it occurs at the block's *natural frequency* for the zero damping state. Resonance can be a desirable feature, as in detecting a particular frequency in a radio frequency detection circuit, or it may be most undesirable, as when a mechanical system resonates, possibly to destruction. It can be seen that it is mostly controlled by the damping ratio. Note also that the phase shift for the second-order system ranges from 0 to 180°. This has important implications if the block is part of a feedback loop because as the frequency rises, the phase shift from the block will pass from stable negative feedback (less than 90°) to positive feedback (greater than 90°), causing unwanted oscillation.

More detail of the various other situations, including how to deal with higher orders, cascaded blocks of similar kind, and ramp inputs are covered elsewhere [1].

3.3 Calibration of Measurements

We have already introduced the concept of accuracy in making a measurement and how the uncertainty inherent in all measurements must be kept sufficiently small. The process and apparatus used to find out if a measurement is accurate enough is called *calibration.* It is achieved by comparing the result of a measurement with a method possessing a measurement performance that is generally agreed to have less uncertainty than that in the result obtained. The error arising within the calibration apparatus and process

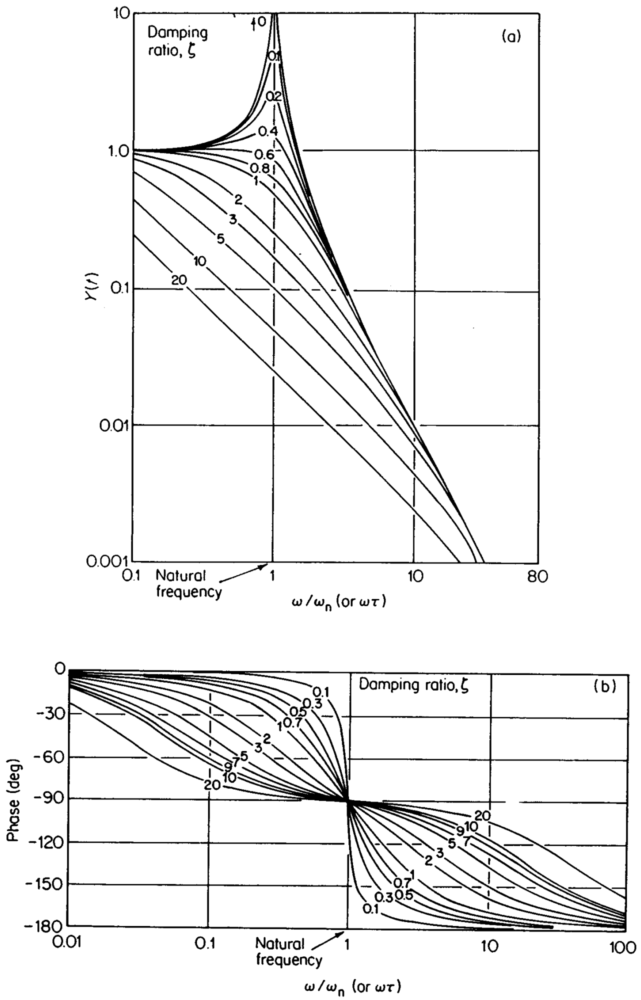

FIGURE 3.17 These two plots allow the behavior of second-order blocks with sine-wave inputs to be ascertained: (*a*) amplitude and (*b*) phase. (From P. H. Sydenham, *Handbook of Measurement Science*, Vol. 2, Chichester, U.K., John Wiley & Sons, 1983. With permission.)

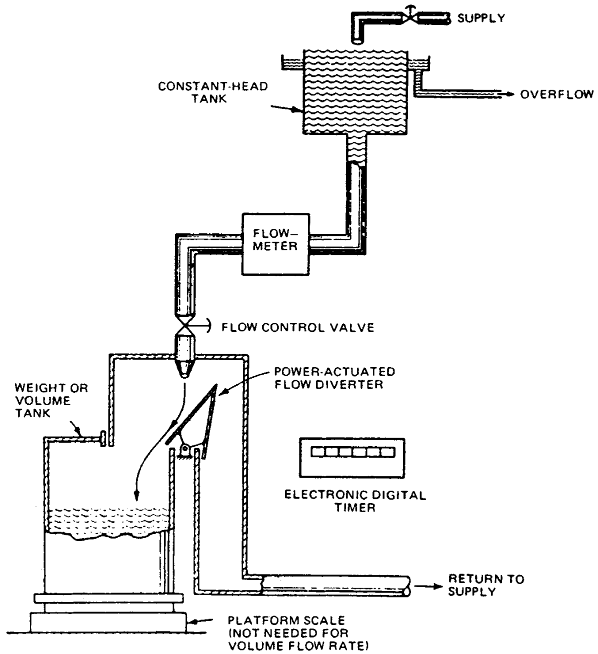

FIGURE 3.18 This practical example illustrates how flow meters are calibrated by passing a known quantity of fluid through the meter over a given time. (Originally published in P. H. Sydenham, *Transducers in Measurement and Control*, Adam Hilger, Bristol, IOP Publishing, Bristol, 1984. Copyright P. H. Sydenham.)

of comparison must necessarily be less than that required. This means that calibration is often an expensive process. Conducting a good calibration requires specialist expertise.

The method and apparatus for performing measurement instrumentation calibrations vary widely. An illustrative example of the comparison concept underlying them all is given in the calibration of flow meters, shown diagrammatically in Figure 3.18.

By the use of an overflowing vessel, the top tank provides a flow of water that remains constant because it comes from a constant height. The meter to be calibrated is placed in the downstream pipe.

The downstream is either deflected into the weigh tank or back to the supply. To make a measurement, the water is first set to flow to the supply. At the start of a test period, the water is rapidly and precisely deflected into the tank. After a given period, the water is again sent back to the supply. This then has filled the tank with a given amount of water for a given time period of flow. Calculations are then undertaken to work out the quantity of water flowing per unit time period, which is the *flow rate*. The meter was already registering a flow rate as a constant value. This is then compared with the weighed method to yield the error. Some thought will soon reveal many sources of error in the test apparatus, such as that the temperature of the water decides the volume that flows through and thus this must be allowed for in the calculations.

It will also be clear that this calibration may not be carried out under the same conditions as the measurements are normally used. The art and science and difficulties inherent in carrying out quality calibration for temperature sensors are well exposed [2].

Calibration of instrumentation is a must for, without it, measurement results may be misleading and lead to costly aftermath situations. Conducting good calibration adds overhead cost to measurement but it is akin to taking out insurance. If that investment is made properly, it will assist in mitigating later penalties. For example, an incorrectly calibrated automatic cement batcher was used in making concrete for the structural frame of a multistory building. It took several days before concrete strength tests revealed the batcher had been out of calibration for a day with the result that the concrete already poured for three floors was not of adequate strength. By then, more stories had been poured on top. The defective floors had to be fully replaced at great cost. More resource put into the calibration process would have ensured that the batcher was working properly.

References

1. P. H. Sydenham, *Handbook of Measurement Science,* Vol. 2, Chichester, U.K.: John Wiley & Sons, 1983.
2. J. V. Nicholas and D. R. White, *Traceable Temperatures,* Chichester, U.K.: John Wiley & Sons, 1994.
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Measurement Accuracy

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All test measurements are taken so that data may be acquired that are useful in decision making. No tests are run and no measurements made when the "answer" is already known. For data to be useful, it is necessary that their measurement errors be small in comparison to the changes or effect under evaluation. Measurement error is unknown and unknowable. This chapter addresses the techniques used to estimate, with some confidence, the expected limits of the measurement errors.

4.1 Error: The Normal Distribution and the Uniform Distribution

Error is defined as the difference between the measured value and the true value of the meaurand [1]. That is,

$$E = (measured) - (true) \tag{4.1}$$

where E = the measurement error
(measured) = the value obtained by a measurement
(true) = the true value of the measurand

It is only possible to estimate, with some confidence, the expected limits of error. The most common method for estimating those limits is to use the *normal distribution* [2]. It is

$$Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(X-\mu)^2/\sigma^2} \tag{4.2}$$

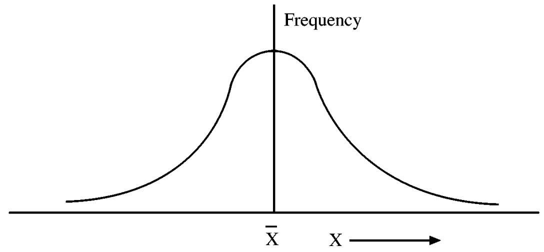

FIGURE 4.1

where X = the input variable, here the value obtained by a measurement
μ = the average of the population of the X variable
σ = the standard deviation of the population, expressed as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \mu)^2}{n}} \tag{4.3}$$

where X_i = the i^{th} X measurement
n = the number of data points measured from the population

Typically, neither n, μ, nor σ are known.

Figure 4.1 illustrates this distribution. Here, for an infinite population ($N = \infty$), the standard deviation, σ, would be used to estimate the expected limits of a particular error with some confidence. That is, the average, plus or minus 2σ divided by the square root of the number of data points, would contain the true average, μ, 95% of the time.

However, in test measurements, one typically cannot sample the entire population and must make do with a sample of N data points. The sample standard deviation, S_X, is then used to estimate σ_X, the expected limits of a particular error. (That sample standard deviation divided by the square root of the number of data points is the starting point for the confidence interval estimate on μ.) For a large dataset (defined as having 30 or more degrees of freedom), plus or minus $2S_X$ divided by the square root of the number of data points in the reported average, M, would contain the true average, μ, 95% of the time. That S_X divided by the square root of the number of data points in the reported average is called the *standard deviation of the average* and is written as:

$$S_{\bar{X}} = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{N-1}} \Big/ \sqrt{M} = S_X \big/ \sqrt{M} \tag{4.4}$$

where $S_{\bar{X}}$ = the standard deviation of the average; the sample standard deviation of the data divided by the square root of M
S_X = the sample standard deviation
$\bar{X}$ = the sample average, that is,

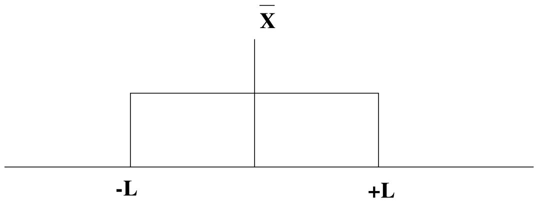

FIGURE 4.2

$$\bar{X} = \sum_{i=1}^{M} \left(X_i / N \right) \tag{4.5}$$

X_i = the i^{th} data point used to calculate the sample standard deviation and the average, $\bar{X}$, from the data

N = the number of data points used to calculate the standard deviation

$(N - 1)$ = the degrees of freedom of S_X and $S_{\bar{X}}$

M = the number of data points in the reported average test result

Note in Equation 4.4 that N does not necessarily equal M . It is possible to obtain S_X from historical data with many degrees of freedom ([$N - 1$] greater than 30) and to run the test only M times. The test result, or average, would therefore be based on M measurements, and the standard deviation of the average would still be calculated with Equation 4.4. In that case, there would be two averages, $\bar{X}$. One $\bar{X}$ would be from the historical data used to calculate the sample standard deviation, and the other $\bar{X}$, the average test result for M measurements.

Note that the sample standard deviation, S_X, is simply:

$$S_X = \sqrt{\frac{\sum_{i=1}^{N} \left(X_i - \bar{X} \right)^2}{N - 1}} \tag{4.6}$$

In some cases, a particular error distribution may be assumed or known to be *a uniform or rectangular distribution*, Figure 4.2, instead of a normal distribution. For those cases, the sample standard deviation of the data is calculated as:

$$S_X = L / \sqrt{3} \tag{4.7}$$

where L = the plus/minus limits of the uniform distribution for a particular error [3].

For those cases, the standard deviation of the average is written as:

$$S_{\bar{X}} = \frac{L / \sqrt{3}}{\sqrt{M}} \tag{4.8}$$

Although the calculation of the sample standard deviation (or its estimation by some other process) is required for measurement uncertainty analysis, all the analytical work computing the measurement uncertainty uses only the standard deviation of the average for each error source.

Uncertainty (Accuracy)

Since the error for any particular error source is unknown and unknowable, its limits, at a given confidence, must be estimated. This estimate is called the *uncertainty.* Sometimes, the term *accuracy* is used to describe the quality of test data. This is the positive statement of the expected limits of the data's errors. Uncertainty is the negative statement. Uncertainty is, however, unambiguous. Accuracy is sometimes ambiguous. (For example,: what is twice the accuracy of ±2%? ±1% or ±4%?) For this reason, this chapter will use the term *uncertainty* throughout to describe the quality of test data.

4.2 Measurement Uncertainty Model

Purpose

One needs an estimate of the uncertainty of test results to make informed decisions. Ideally, the uncertainty of a well-run experiment will be much less than the change or test result expected. In this way, it will be known, with high confidence, that the change or result observed is real or acceptable and not a result of the errors of the test or measurement process. The limits of those errors are estimated with uncertainty, and those error sources and their limit estimators, the uncertainties, may be grouped into classifications to ease their understanding.

Classifying Error and Uncertainty Sources

There are two classification systems in use. The final total uncertainty calculated at a confidence is identical no matter what classification system is used. The two classifications utilized are the *ISO classifications* and the *engineering classifications.* The former groups errors and their uncertainties by type, depending on whether or not there is data available to calculate the sample standard deviation for a particular error and its uncertainty. The latter classification groups errors and their uncertainties by their effect on the experiment or test. That is, the engineering classification groups errors and uncertainties by *random* and *systematic* types, with subscripts used to denote whether there are data to calculate a standard deviation or not for a particular error or uncertainty source. For this reason, engineering classification groups usually are more useful and recommended.

ISO Classifications

This error and uncertainty classification system is not recommended in this chapter, but will yield a total uncertainty in complete agreement with the recommended classification system — the engineering classification system. In this ISO system, errors and uncertainties are classified as Type A if there are data to calculate a sample standard deviation and Type B if there is not [4]. In the latter case, the sample standard deviation might be obtained from experience or manufacturer's specifications, to name two examples.

The impact of multiple sources of error is estimated by root-sum-squaring their corresponding multiple uncertainties. The operating equations are

Type A, data for the calculation of the standard deviation:

$$U_A = \left[\sum_{i=1}^{N_A} \left(\theta_i U_{A_i} \right)^2 \right]^{1/2} \tag{4.9}$$

where U_{A_i} = the standard deviation (based on data) of the average for uncertainty source i of Type A each with its own degrees of freedom. U_A is in units of the test or measurement result. It is an $S_{\bar{X}}$.

N_A = the number of parameters with a Type A uncertainty

θ_i = the sensitivity of the test or measurement result, R, to the i^{th} Type A uncertainty. θ_i is the partial derivative of the result with respect to each i^{th} independent measurement.

The uncertainty of each error source in units of that source, when multiplied by the sensitivity for that source, converts that uncertainty to result units. Then the effect of several error sources may be estimated by root-sum-squaring their uncertainties as they are now all in the same units. The sensitivities, θ_i, are obtained for a measurement result, R, which is a function of several parameters, P_i. The basic equations are

$$R = \text{the measurement result}$$

where $R = f(P_1, P_2, P_3 \ldots P_N)$

P = a measurement parameter used to calculate the result, R

$\theta_i = \partial R / \partial P_i$

Obtaining the θ_i is often called error propagation or uncertainty propagation.

Type B (no data for standard deviation) calculation

$$U_B = \left[\sum_{i=1}^{N_B} \left(\theta_i U_{B_i}\right)^2\right]^{1/2} \tag{4.10}$$

where U_{B_i} = the standard deviation (based on an estimate, not data) of the average for uncertainty source i of Type B; U_B is in units of the test or measurement result, R. It is an $S_{\bar{X}}$

N_B = the number of parameters with a Type B uncertainty

θ_i = the sensitivity of the test or measurement result to the i^{th} Type B uncertainty R

For these uncertainties, it is assumed that the U_{B_i} represent one standard deviation of the average for one uncertainty source with an assumed normal distribution. (They also represent one standard deviation as the square root of the "M" by which they are divided is one, that is, there is only one Type B error sampled from each of these distributions.) The degrees of freedom associated with this standard deviation (also standard deviation of the average) is infinity.

Note that θ_i, the sensitivity of the test or measurement result to the i^{th} Type B uncertainty, is actually the change in the result, R, that would result from a change, of the size of the Type B uncertainty, in the i^{th} input parameter used to calculate that result.

The degrees of freedom of the U_A and the U_{B_i} are needed to compute the degrees of freedom of the combined total uncertainty. It is calculated with the Welch–Satterthwaite approximation. The general formula for degrees of freedom [5] is

$$df_R = \nu_R = \frac{\left[\sum_{i=1}^{N} \left(S_{\bar{X}_i}\right)^2\right]^2}{\left[\sum_{i=1}^{N} \frac{\left(S_{\bar{X}_i}\right)^4}{\left(\nu_i\right)}\right]} \tag{4.11}$$

where $df_R = \nu_R$ = degrees of freedom for the result

ν_i = the degrees of freedom of the i^{th} standard deviation of the average

For the ISO model, Equation 4.11 becomes:

$$df_{R,\mathrm{ISO}} = \nu_{R,\mathrm{ISO}} = \frac{\left[\sum_{i=1}^{N_A} \left(\theta_i U_A\right)^2 + \sum_{i=1}^{N_B} \left(\theta_i U_{B_i}\right)^2\right]^2}{\left[\sum_{i=1}^{N_A} \frac{\left(\theta_i U_{A_i}\right)^4}{\left(\nu_i\right)} + \sum_{i=1}^{N_B} \frac{\left(\theta_i U_{B_i}\right)^4}{\left(\nu_i\right)}\right]} \tag{4.12}$$

The degrees of freedom calculated with Equation 4.12 is often a fraction. This should be truncated to the next lower whole number to be conservative.

Note that in Equations 4.9, 4.10, and 4.12, N_A and N_B need not be equal. They are only the total number of parameters with uncertainty sources of Type A and B, respectively.

In computing a total uncertainty, the uncertainties noted by Equations 4.10 and 4.11 are combined. For the ISO model [3], this is calculated as:

$$U_{R,\mathrm{ISO}} = \pm t_{95}\left[\left(U_A\right)^2 + \left(U_B\right)^2\right]^{1/2} \tag{4.13}$$

where t_{95} = Student's t for ν_R degrees of freedom

Student's t is obtained from Table 4.1.

Note that alternative confidences are permissible. 95% is recommended by the ASME [6], but 99% or 99.7% or any other confidence is obtained by choosing the appropriate Student's t. 95% confidence is, however, recommended for uncertainty analysis.

In all the above, the errors were assumed to be independent. Independent sources of error are those that have no relationship to each other. That is, an error in a measurement from one source cannot be used to predict the magnitude or direction of an error from the other, independent, error source. Nonindependent error sources are related. That is, if it were possible to know the error in a measurement from one source, one could calculate or predict an error magnitude and direction from the other,

TABLE 4.1 Student's t Statistic for 95% Confidence, t_{95}, Degrees of Freedom, ν. This is Frequently Written as: $t_{95,\nu}$

ν	t_{95}	ν	t_{95}	ν	t_{95}
1	12.706	11	2.201	21	2.080
2	4.303	12	2.179	22	2.074
3	3.182	13	2.160	23	2.069
4	2.776	14	2.145	24	2.064
5	2.571	15	2.131	25	2.060
6	2.447	16	2.120	26	2.056
7	2.365	17	2.110	27	2.052
8	2.306	18	2.101	28	2.048
9	2.262	19	2.093	29	2.045
10	2.228	20	2.086	≥30	2.000

nonindependent error source. These are sometimes called *dependent error sources*. Their degree of dependence may be estimated with the linear correlation coefficient. If they are nonindependent, whether Type A or Type B, Equation 4.13 becomes [7]:

$$U_{R,\mathrm{ISO}} = t_{95}\left\{\sum_{T=\mathrm{A}}^{\mathrm{B}}\sum_{i=1}^{N_{i,\mathrm{T}}}\left[\left(\theta_i U_{i,\mathrm{T}}\right)^2 + \sum_{j=1}^{N_{i,\mathrm{T}}}\theta_i\theta_j U_{(i,\mathrm{T}),(j,\mathrm{T})}\left(1-\delta_{i,j}\right)\right]\right\}^{1/2} \tag{4.14}$$

where: $U_{i,\mathrm{T}}$ = the i^{th} elemental uncertainty of Type T (can be Type A or B)
$U_{R,\mathrm{ISO}}$ = the total uncertainty of the measurement or test result
θ_i = the sensitivity of the test or measurement result to the i^{th} Type T uncertainty
θ_j = the sensitivity of the test or measurement result to the j^{th} Type T uncertainty
$U_{(i,\mathrm{T}),(j,\mathrm{T})}$ = the covariance of U_{i,T_i} on $U_{j,\mathrm{T}}$

$$= \sum_{l=1}^{K} U_{i,\mathrm{T}}(l)\, U_{j,\mathrm{T}}(l) \tag{4.15}$$

= the sum of the products of the elemental systematic uncertainties that arise from a common source (l)
l = an index or counter for common uncertainty sources
K = the number of common source pairs of uncertainties
$\delta_{i,j}$ = the Kronecker delta. $\delta_{i,j} = 1$ if $i = j$, and $\delta_{i,j} = 0$ if not [7]
T = an index or counter for the ISO uncertainty type, A or B

This ISO classification equation will yield the same total uncertainty as the engineering classification, but the ISO classification does not provide insight into how to improve an experiment's or test's uncertainty. That is, whether to possibly take more data because the random uncertainties are too high or calibrate better because the systematic uncertainties are too large. The engineering classification now presented is therefore the preferred approach.

Engineering Classification

The engineering classification recognizes that experiments and tests have two major types of errors whose limits are estimated with uncertainties at some chosen confidence. These error types may be grouped as *random* and *systematic*. Their corresponding limit estimators are the random uncertainty and systematic uncertainties, respectively.

Random

The general expression for random uncertainty is the ($1S_{\bar{X}}$) standard deviation of the average [6]:

$$S_{\bar{X},R} = \left[\sum_{T=\mathrm{A}}^{\mathrm{B}}\sum_{i=1}^{N_{i,\mathrm{T}}}\left(\theta_i S_{\bar{X}_{i,\mathrm{T}}}\right)^2\right]^{1/2} = \left[\sum_{T=\mathrm{A}}^{\mathrm{B}}\sum_{i=1}^{N_{i,\mathrm{T}}}\left(\theta_i S_{X_{i,\mathrm{T}}}\Big/\sqrt{M_{i,\mathrm{T}}}\right)^2\right]^{1/2} \tag{4.16}$$

where: $S_{X_{i,\mathrm{T}}}$ = the sample standard deviation of the i^{th} random error source of Type T
$S_{\bar{X}_{i,\mathrm{T}}}$ = the random uncertainty (standard deviation of the average) of the i^{th} parameter random error source of Type T
$S_{\bar{X},R}$ = the random uncertainty of the measurement or test result
$N_{i,T}$ = the total number of random uncertainties, Types A and B, combined
$M_{i,\mathrm{T}}$ = the number of data points averaged for the i^{th} error source, Type A or B
θ_i = the sensitivity of the test or measurement result to the i^{th} random uncertainty

Note that $S_{\bar{X},R}$ is in units of the test or measurement result because of the use of the sensitivities, θ_i. Here, the elemental random uncertainties have been root-sum-squared with due consideration for their sensitivities, or influence coefficients. Since these are all random uncertainties, there is, by definition, no correlation in their corresponding error data so these can always be treated as independent uncertainty sources.

Systematic

The systematic uncertainty of the result, B_R, is the root-sum-square of the elemental systematic uncertainties with due consideration for those that are correlated [7]. The general equation is

$$B_R = \left\{\sum_{T=\mathrm{A}}^{\mathrm{B}}\sum_{i=1}^{N_{\mathrm{T}}}\left[\left(\theta_i B_{i,\mathrm{T}}\right)^2 + \sum_{j=1}^{N_{\mathrm{T}}}\theta_i\theta_j B_{(i,\mathrm{T}),(j,\mathrm{T})}\left(1-\delta_{i,j}\right)\right]\right\}^{1/2} \tag{4.17}$$

where: $B_{i,T}$ = the i^{th} parameter elemental systematic uncertainty of Type T
B_R = the systematic uncertainty of the measurement or test result
N = the total number of systematic uncertainties
θ_i = the sensitivity of the test or measurement result to the i^{th} systematic uncertainty
θ_j = the sensitivity of the test or measurement result to the j^{th} systematic uncertainty
$B_{(i,\mathrm{T}),(j,\mathrm{T})}$ = the covariance of B_i on B_j

$$= \sum_{l=1}^{M} B_{i,\mathrm{T}}(l)\, B_{j,\mathrm{T}}(l) \tag{4.18}$$

= the sum of the products of the elemental systematic uncertainties that arise from a common source (l)
l = an index or counter for common uncertainty sources
$\delta_{i,j}$ = the Kronecker delta. $\delta_{i,j} = 1$ if $i = j$, and $\delta_{i,j} = 0$ if not [7]
T = an index or counter for the ISO uncertainty type, A or B

Here, each $B_{i,T}$ and $B_{j,T}$ are estimated as $2S_X$ for an assumed normal distribution of errors at 95% confidence with infinite degrees of freedom [6].

The random uncertainty, Equation 4.16, and the systematic uncertainty, Equation 4.17, must be combined to obtain a total uncertainty:

$$U_{R,ENG} = t_{95}\left[\left(B_R/2\right)^2 + \left(S_{\bar{X},R}\right)^2\right]^{1/2} \tag{4.19}$$

Note that B_R is in units of the test or measurement result as was $S_{\bar{X},R_i}$.

The degrees of freedom will be needed for the engineering system total uncertainty. It is accomplished with the Welch–Satterthwaite approximation, the general form of which is Equation 4.10, and the specific formulation here is

$$df_R = \nu_R = \frac{\left\{\sum_{T=\mathrm{A}}^{\mathrm{B}}\left[\sum_{i=1}^{N_{S_{\bar{X}_{i,\mathrm{T}}}}}\left(\theta_i S_{\bar{X}_{i,\mathrm{T}}}\right)^2 + \sum_{i=1}^{N_{B_{i,\mathrm{T}}}}\left(\theta_i B_{i,\mathrm{T}/t}\right)^2\right]\right\}^2}{\left\{\sum_{T=\mathrm{A}}^{\mathrm{B}}\left[\sum_{i=1}^{N_{S_{\bar{X}_{i,\mathrm{T}}}}}\frac{\left(\theta_i S_{\bar{X}_{i,\mathrm{T}}}\right)^4}{\left(\nu_{i,\mathrm{T}}\right)} + \sum_{i=1}^{N_{B_{i,\mathrm{T}}}}\frac{\left(\theta_i B_{i,\mathrm{T}/t}\right)^4}{\left(\nu_{i,\mathrm{T}}\right)}\right]\right\}} \tag{4.20}$$

where $N_{S_{\bar{X},T}}$ = the number of random uncertainties of Type T
$N_{B_{i},T}$ = the number of systematic uncertainties of Type T
$\nu_{i,T}$ = the degrees of freedom for the i^{th} uncertainty of Type T
$\nu_{i,T}$ = infinity for all systematic uncertainties
t = Student's t associated with the d.f. for each B_i

Symmetrical Systematic Uncertainties

Most times, all elemental uncertainties will be symmetrical. That is, their ± limits about the measured average will be the same. That is, they will be ±3°C or ±2.05 kPa and the like and not +2.0°C, –1.0°C or, +1.5 kPa, –0.55 kPa. The symmetrical measurement uncertainty may therefore be calculated as follows. (For an elegant treatment of nonsymmetrical uncertainties, see that section in Reference [6].)

Note that throughout these uncertainty calculations, all the uncertainties are expressed in engineering units. All the equations will work with relative units as well. That approach may be seen in Reference [6] also. However, it is often easier to express all the uncertainties and the uncertainty estimation calculations in engineering units and then, at the end, with the total uncertainty, convert the result into relative terms. That is what this section recommends.

4.3 Calculation of Total Uncertainty

ISO Total (Expanded) Uncertainty

The ISO total uncertainty for independent uncertainty sources (the most common) is Equation 4.13:

$$U_{R,\mathrm{ISO}} = \pm t_{95}\left[\left(U_{\mathrm{A}}\right)^2 + \left(U_{\mathrm{B}}\right)^2\right]^{1/2} \quad (4.21)$$

where: $U_{R,\mathrm{ISO}}$ = the measurement uncertainty of the result
U_{A} = the Type A uncertainty for the result
U_{B} = the Type B uncertainty for the result
t_{95} = Student's t_{95} is the recommended multiplier to assure 95% confidence

The ISO uncertainty with some nonindependent uncertainty sources is Equation 4.14:

$$U_{R,\mathrm{ISO}} = \left\{\sum_{T=\mathrm{A}}^{\mathrm{B}}\sum_{i=1}^{N_{i,\mathrm{T}}}\left[\left(\theta_i U_{i,\mathrm{T}}\right)^2 + \sum_{j=1}^{N_{j,\mathrm{T}}}\theta_i\theta_j U_{(i,\mathrm{T}),(j,\mathrm{T})}\left(1-\delta_{i,j}\right)\right]\right\}^{1/2} \quad (4.22)$$

Engineering System Total Uncertainty

The engineering system equation for total uncertainty for independent uncertainty sources (the most common) is

$$U_{R,\mathrm{ENG}} = \pm t_{95}\left[\left(B_R/2\right)^2 + \left(S_{\bar{X},R}\right)^2\right]^{1/2} \quad (4.23)$$

Here, just the first term of Equation 4.23 is needed as all the systematic uncertainty sources are independent.

The engineering system equation for uncertainty for nonindependent uncertainty sources (those with correlated systematic uncertainties) is also Equation 4.23; but remember to use the full expression for B_R, Equation 4.17:

TABLE 4.2 Temperature Measurement Uncertainties, F

Defined measurement process	Systematic uncertainty, B_i	d.f. for B_i	Standard deviation, $S_{\bar{X},i}$	Number of data points, N_i	Random uncertainty, $S_{\bar{X},i}$	Degrees of freedom, d.f., ν_i
Calibration of tc	0.06_{B}	∞	0.3_{A}	10	0.095_{A}	9
Reference junction	0.07_{A}	12	0.1_{A}	5	0.045_{A}	4
Data acquisition	0.10_{B}	∞	0.6_{A}	12	0.173_{A}	11
RSS						

$$B_R = \left\{\sum_{T=\mathrm{A}}^{\mathrm{B}}\sum_{i=1}^{N_{\mathrm{T}}}\left[\left(\theta_i B_{i,\mathrm{T}}\right)^2 + \sum_{j=1}^{N_{\mathrm{T}}}\theta_i\theta_j B_{(i,\mathrm{T}),(j,\mathrm{T})}\left(1-\delta_{i,j}\right)\right]\right\}^{1/2} \quad (4.24)$$

The degrees of freedom for Equations 4.21 through 4.24 is calculated with the Welch–Satterthwaite approximation, Equation 4.12 for the ISO system and Equation 4.20 for the engineering system.

High Degrees of Freedom Approximation

It is often the case that it is assumed that the degrees of freedom are 30 or higher. In these cases, the equations for uncertainty simplify further by setting t_{95} equal to 2.000. This approach is recommended for a first-time user of uncertainty analysis procedures as it is a fast way to get to an approximation of the measurement uncertainty.

Calculation Example

The following calculation example is taken where all the uncertainties are independent and are in the units of the test result — temperature. It is a simple example that illustrates the combination of measurement uncertainties in their most basic case. More detailed examples are given in many of the references cited. Their review may be needed to assure a more comprehensive understanding of uncertainty analysis.

It has been shown [8] that there is often little difference in the uncertainties calculated with the different models. The data from Table 4.2 [9] will be used to calculate measurement uncertainty with these two models. These data are all in temperature units and thus the influence coefficients, or sensitivities, are all unity.

Note the use of subscripts "A" and "B" to denote where data exist to calculate a standard deviation. Note too that in this example, all errors (and therefore uncertainties) are independent and that all degrees of freedom for the systematic uncertainties are infinity except for the reference junction whose degrees of freedom are 12. Also note that B_R is calculated as:

$$B_R = 2\left[\left(\frac{0.06}{2}\right)^2 + \left(\frac{0.07}{2.18}\right)^2 + \left(\frac{0.1}{2}\right)^2\right]^{1/2} = 0.13 \quad (4.25)$$

Each uncertainty model will now be used to derive a measurement uncertainty.

For the U_{ISO} model one obtains, via Equation 4.13, the expression:

$$U_A = \left[\left(0.095\right)^2 + \left(0.045\right)^2 + \left(0.173\right)^2 + \left(\frac{0.07}{2.18}\right)^2\right]^{1/2} = 0.21 \quad (4.27)$$

$$U_B = \left[\left(\frac{0.06}{2}\right)^2 + \left(\frac{0.10}{2}\right)^2\right]^{1/2} = 0.058 \tag{4.28}$$

$$U_{R,\mathrm{ISO}} = \pm K\left[\left(U_A\right)^2 + \left(U_B\right)^2\right]^{1/2} = \pm K\left[\left(0.21\right)^2 + \left(0.058\right)^2\right]^{1/2} \tag{4.29}$$

Here, remember that the 0.21 is the root sum square of the $1S_{\bar{X}}$ Type A uncertainties in Table 4.2, and 0.058 that for the $1S_{\bar{X}}$ Type B uncertainties. Also note that in most cases, the Type B uncertainties have infinite degrees of freedom and represent an equivalent $2S_X$. That is why they are divided by 2 — to get an equivalent $1S_X$. Where there are less than 30 degrees of freedom, one needs to divide by the appropriate Student's *t* that gave the 95% confidence interval. For the reference junction systematic uncertainty above, that was 2.18.

If "*K*" is taken as Student's t_{95}, the degrees of freedom must first be calculated. Remember that all the systematic components of Type "B" have infinite degrees of freedom except for the 0.07, which has 12 degrees of freedom. Also, all the B_i in Table 4.1 represent an equivalent $2S_X$ except for 0.07, which represents $2.18S_X$, as its degrees of freedom are 12 and not infinity. To use their data here, divide them all but the 0.07 by 2 and the 0.07 by 2.18 so they all now represent $1S_X$, as do the random components. All Type A uncertainties, whether systematic or random in Table 4.1, have degrees of freedom as noted in the table. The degrees of freedom for U_{ISO} is then:

$$df_R = \nu_R = \frac{\left[\left(0.095\right)^2 + \left(0.045\right)^2 + \left(0.173\right)^2 + \left(0.06/2\right)^2 + \left(0.07/2.18\right)^2 + \left(0.10/2\right)^2\right]^2}{\left[\frac{\left(0.095\right)^4}{9} + \frac{\left(0.045\right)^4}{4} + \frac{\left(0.173\right)^4}{11} + \frac{\left(0.06\right)^4}{\infty} + \frac{\left(0.07/2.18\right)^4}{12} + \frac{\left(0.10/2\right)^4}{\infty}\right]} = 22.51 \approx 22 \tag{4.30}$$

t_{95} is therefore 2.07. $U_{R,\mathrm{ISO}}$ is then:

$$U_{R,\mathrm{ISO}} = \pm 2.07\left[\left(0.21\right)^2 + \left(0.058\right)^2\right]^{1/2} = 0.45 \text{ for 95\% confidence} \tag{4.31}$$

For a detailed comparison to the engineering system, here denoted as the $U_{R,\mathrm{ENG}}$ model, three significant figures are carried so as not to be affected by round-off errors. Then:

$$U_{R,\mathrm{ISO}} = \pm 2.074\left[\left(0.205\right)^2 + \left(0.0583\right)^2\right]^{1/2} = 0.442 \text{ for 95\% confidence} \tag{4.32}$$

For the engineering system, $U_{R,\mathrm{ENG}}$, model, Equation 4.23, one obtains the expression:

$$U_{R,\mathrm{ENG}} = \pm t_{95}\left[\left(0.13/2\right)^2 + \left(0.20\right)^2\right]^{1/2} \tag{4.33}$$

Here, the (0.13/2) is the $B_R/2$ and the 0.20 is as before the random component. To obtain the proper t_{95}, the degrees of freedom need to be calculated just as in Equation 4.30. There, the degrees of freedom were 22 and t_{95} equals 2.07. $U_{R,\mathrm{ENG}}$ is then:

$$U_{R,\mathrm{ENG}} = \pm 2.07\left[\left(0.13/2\right)^2 + \left(0.20\right)^2\right]^{1/2} = 0.44 \text{ for 95\% confidence} \tag{4.34}$$

Carrying four significant figures for a comparison to $U_{R,ISO}$ not affected by round-off errors, one obtains:

$$U_{R,\mathrm{ENG}} = \pm 2.074\left[\left(0.133/2\right)^2 + \left(0.202\right)^2\right]^{1/2} = 0.442 \text{ for 95\% confidence} \tag{4.35}$$

This is identical to $U_{R,\mathrm{ISO}}$, Equation 4.32, as predicted.

4.4 Summary

Although these formulae for uncertainty calculations will not handle every conceivable situation, they will provide, for most experimenters, a useful estimate of test or measurement uncertainty. For more detailed treatment or specific applications of these principles, consult the references and the recommended "Further Information" section at the end of this chapter.

Defining Terms

Accuracy: The antithesis of uncertainty. An expression of the maximum possible limit of error at a defined confidence.

Confidence: A statistical expression of percent likelihood.

Correlation: The relationship between two datasets. It is not necessarily evidence of cause and effect.

Degrees of freedom: The amount of room left for error. It may also be expressed as the number of independent opportunities for error contributions to the composite error.

Error: [Error] = [Measured] – [True]. It is the difference between the measured value and the true value.

Influence coefficient: *See* sensitivity.

Measurement uncertainty: The maximum possible error, at a specified confidence, that may reasonably occur. Errors larger than the measurement uncertainty should rarely occur.

Non-symmetrical uncertainty: An uncertainty for which there is an uneven likelihood that the true value lies on one side of the average or the other.

Propagation of uncertainty: An analytical technique for evaluating the impact of an error source (and its uncertainty) on the test result. It employs the use of influence coefficients.

Random error: An error that causes scatter in the test result.

Random uncertainty: An estimate of the limits of random error, usually one standard deviation of the average.

Sensitivity: An expression of the influence an error source has on a test or measured result. It is the ratio of the change in the result to an incremental change in an input variable or parameter measured.

Standard deviation of the average or mean: The standard deviation of the data divided by the number of measurements in the average.

Systematic error: An error that is constant for the duration of a test or measurement.

Systematic uncertainty: An estimate of the limits of systematic error, usually taken as 95% confidence for an assumed normal error distribution.

True value: The desired result of an experimental measurement.

Welch–Satterthwaite: The approximation method for determining the number of degrees of freedom in the random uncertainty of a result.

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Measurement Standards

DeWayne B. Sharp
Shape of Things

Measurement standards are those devices, artifacts, procedures, instruments, systems, protocols, or processes that are used to define (or to realize) measurement units and on which all lower echelon (less accurate) measurements depend. A measurement standard may also be said to store, embody, or otherwise provide a physical quantity that serves as the basis for the measurement of the quantity. Another definition of a standard is the physical embodiment of a measurement unit, by which its assigned value is defined, and to which it can be compared for calibration purposes. In general, it is not independent of physical environmental conditions, and it is a true embodiment of the unit only under specified conditions. Another definition of a standard is a unit of known quantity or dimension to which other measurement units can be compared.

5.1 A Historical Perspective

Many early standards were based on the human body: the length of man's hand, the width of his thumb, the distance between outstretched fingertips, the length of one's foot, a certain number of paces, etc. In the beginning, while groups were small, such standards were convenient and uniform enough to serve as the basis for measurements.

The logical person to impose a single standard was the ruler of the country — hence, our own 12-inch or other short measuring stick is still called a *ruler*. The establishment of measurement standards thus became the prerogative of the king or emperor, and this right has since been assumed by all governments.

History is replete with examples that show the importance of measurements and standards. In a report to the U.S. Congress in 1821, John Quincy Adams said, "Weights and measures may be ranked among the necessaries to every individual of human society." Our founding fathers thought them so important that the United States Constitution expressly gives the Congress the power to fix uniform standards of weights and measures. The need for weights and measures (standards) dates back to earliest recorded history and are even mentioned in the Old Testament of the Bible. Originally, they were locally decreed

to serve the parochial needs of commerce, trade, land division, and taxation. Because the standards were defined by local or regional authorities, differences arose that often caused problems in commerce and early scientific investigation. The rapid growth of science in the late 17th century highlighted a number of serious deficiencies in the system of units then in use and, in 1790, led the French National Assembly to direct the French Academy of Sciences to "deduce an invariable standard for all measures and all the weights." The Academy proposed a system of units, the metric system, to define the unit of length in terms of the earth's circumference, with the units of volume and mass being derived from the unit of length. Additionally, they proposed that all multiples of each unit be a multiple of 10.

In 1875, the U.S. and 16 other countries signed the "Treaty of the Meter," establishing a common set of units of measure. It also established an International Bureau of Weights and Measures (called the BIPM). That bureau is located in the Parisian suburb of Sèvres. It serves as the worldwide repository of all the units that maintain our complex international system of weights and measures. It is through this system that compatibility between measurements made thousands of miles apart is currently maintained.

The system of units set up by the BIPM is based on the meter and kilogram instead of the yard and the pound. It is called the Système International d'Unités (SI) or the International System of Units. It is used in almost all scientific work in the U.S. and is the only system of measurement units in most countries of the world today.

Even a common system of units does not guarantee measurement agreement, however. Therein lies the crux of the problem. We must make measurements, and we must know how accurately (or, to be more correct, with what uncertainty) we made those measurements. In order to know that, there must be standards. Even more important, everyone must agree on the values of those standards and use the same standards.

As the level of scientific sophistication improved, the basis for the measurement system changed dramatically. The earliest standards were based on the human body, and then attempts were made to base them on "natural" phenomena. At one time, the basis for length was supposed to be a fraction of the circumference of the earth but it was "maintained" by the use of a platinum/iridium bar. Time was maintained by a pendulum clock but was defined as a fraction of the day and so on. Today, the meter is no longer defined by an artifact. Now, the meter is the distance that light travels in an exactly defined fraction of a second. Since the speed of light in a vacuum is now defined as a constant of nature with a specified numerical value (299, 792, 458 m/s), the definition of the unit of length is no longer independent of the definition of the unit of time.

Prior to 1960, the second was defined as 1/86,400th of a mean solar day. Between 1960 and 1967, the second was defined in terms of the unit of time implicit in the calculation of the ephemerides: "The second is the fraction 1/31, 556, 925.9747 of the tropical year for January 0 at 12 hours of ephemeris time." With the advent of crystal oscillators and, later, atomic clocks, better ways were found of defining the second. This, in turn, allowed a better understanding of things about natural phenomena that would not have been possible before. For example, it is now known that the earth does not rotate on its axis in a uniform manner. In fact, it is erratically slowing down. Since the second is maintained by atomic clocks it is necessary to add "leap seconds" periodically so that the solar day does not gradually change with respect to the time used every day. It was decided that a constant frequency standard was preferred over a constant length of the day.

5.2 What Are Standards?

One problem with standards is that there are several kinds. In addition to "measurement standards," there are "standards of practice or protocol standards" that are produced by the various standards bodies such as the International Organization for Standardization (ISO), the International Electrotechnical Commission (IEC), the American National Standards Institute (ANSI), and the Standards Council of Canada (SCC). See Figure 5.1.

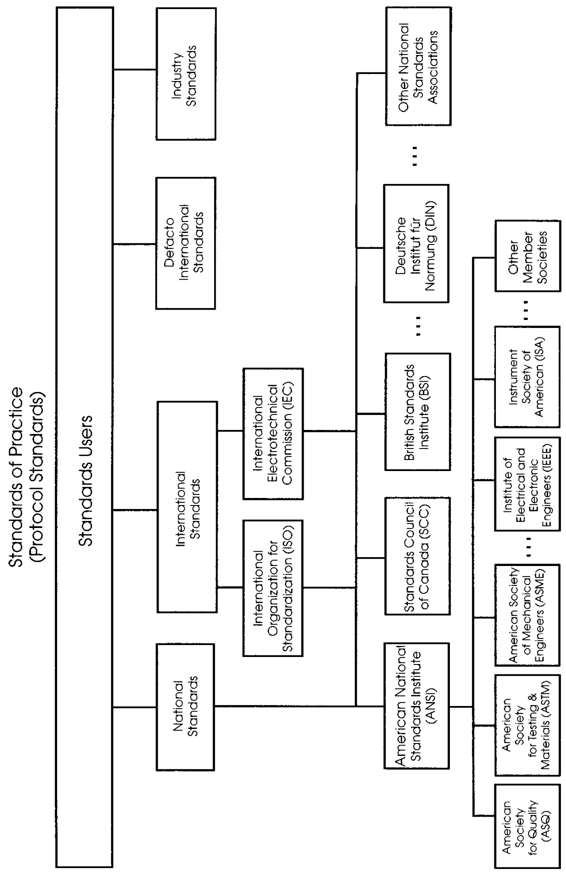

FIGURE 5.1

Standards of Practice (Protocol Standards)

These standards define everything from the dimensions and electrical characteristics of a flashlight battery to the shape of the threads on a machine screw and from the size and shape of an IBM punched card to the Quality Assurance Requirements for Measuring Equipment. Such standards can be defined as documents describing the operations and processes that must be performed in order for a particular end to be achieved. They are called a "protocol" by Europeans to avoid confusion with a physical standard.

Legal Metrology

The application of measurement standards to the control of the daily transactions of trade and commerce is known as Legal Metrology; within the U.S., it is more commonly known as Weights and Measures. Internationally, coordination among nations on Legal Metrology matters is, by international agreement, handled by a quasi-official body — the International Organization for Legal Metrology (OIML).

Within the U.S., domestic uniformity in legal metrology matters is the responsibility of National Institute of Standards and Technology (NIST) acting through its Office of Weights and Measures. Actual enforcement is the responsibility of each of the 50 states and the various territories. These, in turn, generally delegate the enforcement powers downward to their counties and, in some cases, to large cities.

Forensic Metrology

Forensic Metrology is the application of measurements and hence measurement standards to the solution and prevention of crime. It is practiced within the laboratories of law enforcement agencies throughout the world. Worldwide activities in Forensic Metrology are coordinated by Interpol (*Inter*national *Pol*ice; the international agency that coordinates the police activities of the member nations). Within the U.S., the Federal Bureau of Investigation (FBI), an agency of the Department of Justice, is the focal point for most U.S. forensic metrology activities.

Standard Reference Materials

Another type of standard that should be mentioned here are Standard Reference Materials (SRM). Standard Reference Materials are discrete quantities of substances or minor artifacts that have been certified as to their composition, purity, concentration, or some other characteristic useful in the calibration of the measurement devices and the measurement processes normally used in the process control of those substances. SRMs are the essential calibration standards in stoichiometry (the metrology of chemistry).

In the U.S., the National Institute of Standards and Technology (NIST), through its Standard Reference Materials Program, offers for sale over 1300 SRMs. These range from ores to pure metals and alloys. They also include many types of gases and gas mixtures; and many biochemical substances and organic compounds. Among the artifact devices available are optical filters with precise characteristics and standard lamps with known emission characteristics.

5.3 A Conceptual Basis of Measurements

Lord Kelvin's oft-quoted statement may bear repeating here:

> I often say that when you can measure what you are speaking about, and can express it in numbers, you know something about it; but when you cannot measure it, cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginnings of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be. So therefore, if science is measurement, then without metrology there can be no science.

William Thomson (Lord Kelvin), May 6, 1886

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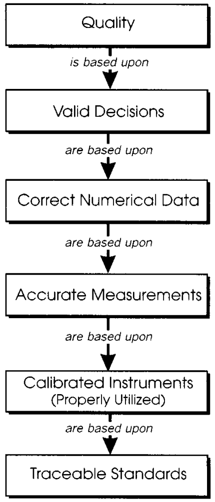

FIGURE 5.2

Lord Kelvin's statement has been quoted so many times that it has almost become trite, but looking at Figure 5.2 will show an interesting hierarchy. In order to achieve quality or "to do things right," it is necessary to make some decisions. The correct decisions cannot be made unless there are good numerical data on which to base those decisions. Those numerical data, in turn, must come from measurements and if "correct" decisions are really needed, they must be based on the "right" numbers. The only way to get "good" numerical data is to make accurate measurements using calibrated instruments that have been properly utilized. Finally, if it is important to compare those measurements to other measurements made at other places and other times, the instruments must be calibrated using traceable standards.

5.4 The Need for Standards

Standards define the units and scales in use, and allow comparison of measurements made in different times and places. For example, buyers of fuel oil are charged by a unit of liquid volume. In the U.S., this would be the gallon; but in most other parts of the world, it would be the liter. It is important for the buyer that the quantity ordered is actually received and the refiner expects to be paid for the quantity shipped. Both parties are interested in accurate measurements of the volume and, therefore, need to agree on the units, conditions, and method(s) of measurement to be used.

Persons needing to measure a mass cannot borrow the primary standard maintained in France or even the national standard from the National Institute of Standards and Technology (NIST) in the U.S. They must use lower-level standards that can be checked against those national or international standards. Everyday measuring devices, such as scales and balances, can be checked (calibrated) against working level mass standards from time to time to verify their accuracy. These working-level standards are, in turn, calibrated against higher-level mass standards. This chain of calibrations or checking is called "traceability." A proper chain of traceability must include a statement of uncertainty at every step.

5.5 Types of Standards

Basic or Fundamental Standards

In the SI system, there are seven basic measurement units from which all other units are derived. All of the units except one are defined in terms of their unitary value. The one exception is the unit of mass. It is defined as 1000 grams (g) or 1 kilogram (kg). It is also unique in that it is the only unit currently based on an artifact. The U.S. kilogram and hence all other standards of mass are based on one particular platinum/iridium cylinder kept at the BIPM in France. If that International Prototype Kilogram were to change, all other mass standards throughout the world would be wrong.

The seven basic units are listed in Appendix 1, Table 1. Their definitions are listed in Appendix 1, Table 2.

Derived Standards

All of the other units are derived from the seven basic units described in Appendix 1, Table 1. Measurement standards are devices that represent the SI standard unit in a measurement. (For example, one might use a zener diode together with a reference amplifier and a power source to supply a known voltage to calibrate a digital voltmeter. This could serve as a measurement standard for voltage and be used as a reference in a measurement.)

Appendix 1, Table 3 lists the most common derived SI units, together with the base units that are used to define the derived unit. For example, the unit of frequency is the hertz; it is defined as the reciprocal of time. That is, 1 hertz (1 Hz) is one cycle per second.

The Measurement Assurance System

Figure 5.3 illustrates the interrelationship of the various categories of standards throughout the world. While it gives more detail to U.S. structure, similar structures exist in other nations. Indeed, a variety of regional organizations exist that help relate measurements made in different parts of the world to each other.

5.6 Numbers, Dimensions, and Units

A measurement is always expressed as a multiple (or submultiple) of some unit quantity. That is, both a numeric value and a unit are required. If electric current were the measured quantity, it might be expressed as some number of milliamperes or even microamperes. It is easy to take for granted the existence of the units used, because their names form an indispensable part of the vocabulary.

5.7 Multiplication Factors

Since it is inconvenient to use whole units in many cases, a set of multiplication factors has been defined that can be used in conjunction with the units to bring a value being measured to a more reasonable size. It would be difficult to have to refer to large distances in terms of the meter; thus, one defines longer distances in terms of kilometers. Short distances are stated in terms of millimeters, micrometers, nanometers, etc. See Appendix 1, Table 4.

Defining Terms

Most of the definitions in this listing were taken from the *International Vocabulary of Basic and General Terms in Metrology*, published by the ISO, 1993 (VIM) [7]. They are indicated by the inclusion (in brackets) of their number designation in the VIM. The remainder of the definitions are not intended to

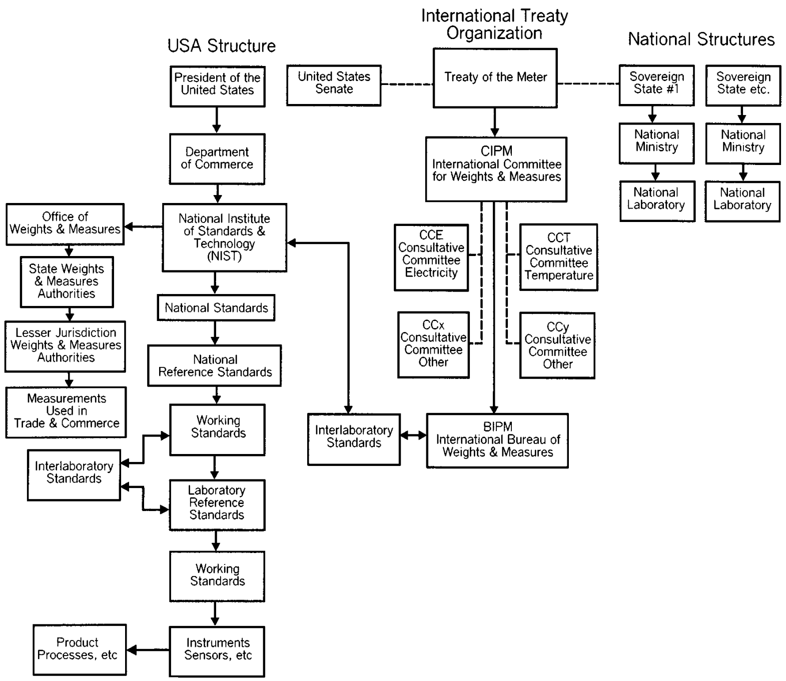

FIGURE 5.3

represent any official agency but are ones widely accepted and are included to help in the understanding of this material. More detailed and rigorous definitions can be found in other works available from ANSI, IEC, ISO, and NIST. Words enclosed in parentheses "(…)" may be omitted from the term if it is unlikely that such omission will cause confusion.

Accuracy of measurement [3.5]:* The closeness of the agreement between the result of a measurement and a true value of the measurand.

NOTES:

1. *Accuracy* is a qualitative concept.
2. The term *precision* should not be used for *accuracy.* (Precision only implies repeatability.)

Note, that to say an instrument is accurate to 5% (a common way of stating it) is wrong. One would not find such an instrument very useful if it, in fact, were only accurate 5% of the time. What is meant when such a statement is made is that the instrument's inaccuracy is less than 5% and it is accurate to better than 95%. Unfortunately, this statement is almost as imprecise as "accurate to 5%." An instrument would not be useful if it were accurate only 95% of the time; but this is not what is implied by "5% accuracy." What is meant is that, (almost) all of the time, its indication is within 5% of the "true" value.

Calibration [6.11]: A set of operations that establish, under specified conditions, the relationship between values of quantities indicated by a measuring instrument or measuring system, or values represented by a material measure or a reference material, and the corresponding values realized by standards.

NOTES:

1. The result of a calibration permits either the assignment of values of measurands to the indicators or the determination of corrections with respect to indications.
2. A calibration can also determine other metrological properties, such as the effect of influence quantities.
3. The result of a calibration can be recorded in a document, sometimes called a *calibration certificate* or a *calibration report.*

Calibration Laboratory: A work space, provided with test equipment, controlled environment and trained personnel, established for the purpose of maintaining proper operation and accuracy of measuring and test equipment. *Calibration laboratories* typically perform many routine calibrations, often on a production-line basis.

Certified Reference Material (CRM) [6.14]: A *reference material*, accompanied by a certificate, one or more of whose property values are certified by a procedure that established traceability to an accurate realization of the unit in which the property values are expressed, and for which each certified value is accompanied by an uncertainty at a stated level of confidence.

1. The definition of a *reference material* is given elsewhere in this vocabulary.
2. CRMs are generally prepared in batches for which the property values are determined within stated uncertainty limits by measurements on samples representative of the entire batch.
3. The certified properties of certified reference materials are sometimes conveniently and reliably realized when the material is incorporated into a specifically fabricated device, e.g., a substance of known triple-point into a triple-point cell, a glass of known optical density into a transmission filter, spheres of uniform particle size mounted on a microscope slide. Such devices can also be considered CRMs.
4. All CRMs lie within the definition of "measurement standards" given in the International Vocabulary of basic and general terms in metrology (VIM).
5. Some RMs and CRMs have properties that, because they cannot be correlated with an established chemical structure or for other reasons, cannot be determined by exactly defined physical and chemical measurement methods. Such materials include certain biological materials such as vaccines to which an International unit has been assigned by the World Health Organization.

This definition, including the Notes, is taken from ISO Guide 30:1992.

Coherent (derived) unit (of measurement) [1.10]: A derived unit of measurement that may be expressed as a product of powers of base units with the proportionality factor one (1).

NOTE: Coherency can be determined only with respect to the base units of a particular system. A unit can be coherent with respect to one system but not to another.

Coherent system of units (of measurement) [1.11]: A system of units of measurement in which all of the derived units are coherent.

Conservation of a (measurement) standard [6.12]: A set of operations, necessary to preserve the metrological characteristics of a measurement standard within appropriate limits.

NOTE: The operations commonly include periodic calibration, storage under suitable conditions, and care in use.

Interlaboratory Standard: A device that travels between laboratories for the sole purpose of relating the magnitude of the physical unit represented by the standards maintained in the respective laboratories.

International (measurement) standard [6.2]: A standard recognized by an international agreement to serve internationally as the basis for assigning values to other standards of the quantity concerned.

International System of Units (SI) [1.12]: The coherent system of units adopted and recommended by the General Conference on Weights and Measures (CGPM).

NOTE: The SI is based at present on the following seven base units: meter, kilogram, second, ampere, kelvin, mole, and candela.

Measurand [2.6]: A particular quantity subject to measurement.

EXAMPLE: Vapor pressure of a given sample of water at 20°C.

NOTE: The specification of a measurand may require statements about quantities such as time, temperature, and pressure.

Measurement [2.1]: A set of operations having the object of determining a value of a quantity.

NOTE: The operations may be performed automatically.

Method of Measurement [2.4]: A logical sequence of operations, described generically, used in the performance of measurements.

NOTE: Methods of measurement may be qualified in various ways, such as:

- Substitution method
- Differential method
- Null method

Metrology [2.2]: The science of measurement.

NOTE: Metrology includes all aspects, both theoretical and practical, with reference to measurements, whatever their uncertainty, and in whatever fields of science or technology they occur.

National (measurement) Standard [6.3]: A standard recognized by a national decision to serve, in a country, as the basis for assigning values to other standards of the quantity concerned.

National Reference Standard: A standard maintained by national laboratories such as the National Institute of Standards and Technology (NIST) in Gaithersburg, MD; the National Research Council (NRC) located in Ottawa, Canada; the National Physical Laboratory (NPL) in Teddington, U.K.; the Physikalisch-Technische Bundesanstalt (PTB) at Braunschweig, Germany; and which are the legal standards of their respective countries.

National Institute of Standards and Technology (NIST): The U.S. national standards laboratory, responsible for maintaining the physical standards upon which measurements in the U.S. are based.

Primary Standard [6.4]: A standard that is designated or widely acknowledged as having the highest metrological qualities and whose value is accepted without reference to other standards of the same quantity.

NOTE: The concept of primary standard is equally valid for base quantities and derived quantities.

Principle of Measurement [2.3]: The scientific base of a measurement.

EXAMPLES:

- The thermoelectric effect applied to the measurement of temperature
- The Josephson effect applied to the measurement of electric potential difference
- The Doppler effect applied to the measurement of velocity
- The Raman effect applied to the measurement of the wave number of molecular vibrations

Reference Standard [6.6]: A standard, generally having the highest metrological quality available at a given location or in a given organization, from which measurements made there are derived.

Reference Material [6.13]: A material or substance, one or more of whose property values are sufficiently homogeneous and well established to be used for the calibration of an apparatus, the assessment of a measurement method, or for assigning values to materials.

NOTE: A reference material can be in the form of a pure or mixed gas, liquid or solid. Examples are water for the calibration of viscometers, sapphire as a heat-capacity calibrant in calorimetry, and solutions used for calibration in chemical analysis.

This definition, including the Note, is taken from ISO Guide 30:1992.

Repeatability (of results of measurements) [3.6]: The closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement.

NOTES:

1. These conditions are called *repeatability conditions.*
2. Repeatability conditions include:
   a. The same measurement process
   b. The same observer
   c. The same measuring instrument, used under the same conditions
   d. The same location
   e. Repetition over a short period of time
3. Repeatability can be expressed quantitatively in terms of the dispersion of characteristics of the results.

Reproducibility (of results of measurements) [3.7]: The closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement.

NOTES:

1. A valid statement of reproducibility requires specification of the conditions changed.
2. The changed conditions include:
   a. Principle of measurement
   b. Method of measurement
   c. Observer
   d. Measuring instrument
   e. Reference standard
   f. Location
   g. Condition of use
   h. Time
3. Reproducibility can be expressed quantitatively in terms of the dispersion characteristics of the results.
4. Results here are usually understood to be corrected results.

Secondary Standard [6.5]: A standard whose value is assigned by comparison with a primary standard of the same quantity.

Standards Laboratory: A work space, provided with equipment and standards, a properly controlled environment, and trained personnel, established for the purpose of maintaining traceability of standards and measuring equipment used by the organization it supports. Standards laboratories typically perform fewer, more specialized and higher accuracy measurements than Calibration Laboratories.

Tolerance: In metrology, the limits of the range of values (the uncertainty) that apply to a properly functioning measuring instrument.

Traceability [**6.10**]: The property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties.

NOTE:

1. The concept is often expressed by the adjective *traceable.*
2. The unbroken chain of comparisons is called a *traceability chain.*

Even though the ISO has published (and accepted) the definition listed above, many practitioners endeavor to make this term more meaningful. They feel that the definition should introduce the aspect of evidence being presented on a continuing basis, to overcome the idea that if valid traceability is achieved, it could last forever. A definition similar to the following one would meet that requirement.

Traceability is a characteristic of a calibration or a measurement. A traceable measurement or calibration is achieved only when each instrument and standard, in a hierarchy stretching back to the national (or international) standard was itself properly calibrated and the results properly documented including statements of uncertainty on a continuing basis. The documentation must provide the information needed to show that all the calibrations in the chain of calibrations were appropriately performed.

Transfer Standard [**6.8**]: A standard used as an intermediary to compare standards.

NOTE: The term *transfer device* should be used when the intermediary is not a standard.

Traveling Standard [**6.9**]: A standard, sometimes of special construction, intended for transport between locations.

EXAMPLE: A portable battery-operated cesium frequency standard.

Uncertainty of Measurement [**3.9**]: A parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

NOTES:

1. The parameter can be, for example, a standard deviation (or a given multiple of it), or the half-width of an interval having a stated level of confidence.
2. Uncertainty of measurement comprises, in general, many components. Some of these components can be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which can also be characterized by standard deviations, are evaluated from assumed probability distributions based on experience or other information.
3. It is understood that the result of the measurement is the best estimate of the value of the measurand, and that all components of uncertainty (including those arising from systematic effects) such as components associated with corrections and reference standards, contribute to the dispersion.

This definition is that of the *Guide to the Expression of Uncertainty in Measurement,* in which its rationale is detailed (see, in particular, 2.2.4 and annex D).[4]

Value (of a quantity) [**1.18**]: The magnitude of a particular quantity generally expressed as a unit of measurement multiplied by a number.

EXAMPLES:

- Length of a rod: 5.34 m or 534 cm
- Mass of a body: 0.152 kg or 152 g
- Amount of substance of a sample of water (H_2O): 0.012 mol or 12 mmol

NOTES:

1. The value of a quantity can be positive, negative, or zero.
2. The value of a quantity can be expressed in more than one way.

3. The values of quantities of dimension one are generally expressed as pure numbers.
4. A quantity that cannot be expressed as a unit of measurement multiplied by a number can be expressed by reference to a conventional reference scale or to a measurement procedure or both.

Working Standard [**6.7**]: A standard that is used routinely to calibrated or check material measures, measuring instruments or reference materials.

NOTES:

1. A working standard is usually calibrated against a *reference standard.*
2. A working standard used routinely to ensure that a measurement is being carried out correctly is called a *check standard.*

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